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What are the equilibria in linear public-good experiments?

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Abstract:
Most social-preference models have been tailored to yield only a full-defection equilibrium in one-shot linear public-good situations. This paper determines the Nash-equilibrium sets that result from experiment participants' elicited preferences. The data show that multiple equilibria are relatively frequent even in a standard three-player setting. In this perspective, the common finding of close-to-omnilateral defection at the end of repeated public-good games is surprising and raises the question of why the dynamics of play seem to select this equilibrium out of the existing equilibria.

Keywords: Public good, social dilemma, Nash-equilibrium, conditional cooperation, social preferences.
JEL: C72, C92, D83, H41

1 Introduction

In many social-game protocols, ranging from gift-exchange over ultimatum bargaining to public-good situations, human behaviour differs substantially from the Nash-equilibrium that results if we assume that players care only about their own monetary payoff.[1] To resolve this discrepancy, numerous models of social preferences have been proposed.[2] These models have been tailored to fit stylised facts from the laboratory. To take a prominent example, Fehr and Schmidt (1999) took it as support for their model that it singles out full-defection as the virtually-unique equilibrium in the typical public-good setting, given most experiment participants cease to contribute in the final round(s) of repeated-play experiments. Yet, no

[1] E.g., Berg et al. (1995), Fehr et al. (1993), Güth et al. (1982), or the papers reviewed in Ledyard (1995).

[2] E.g., Rabin (1993), Fehr and Schmidt (1999), or Levine (1998).

study has documented empirically how often omnilateral defection is an equilibrium given participants' preferences, and how often there are additional, positive-contributions equilibria. This is what the present paper does.

This paper elicits preferences under three different sets of parameters of a linear public-good protocol, and documents the Nash equilibria of all potential within-treatment matchings of participants, which I call *revealed-preference Nash-equilibria* (RPNE). Thereby, I provide empirical evidence of the strategic environment induced by different public-good protocols that can be contrasted with theory.[3] For example, under the parameters used in the experiment, the model by Fehr and Schmidt (1999) predicts multiple equilibria in only 6% of all randomly-formed groups in my three-player setup, which contrasts with an observed 38%. Treatment variations provide evidence on how the observed equilibrium sets vary with changes in the experimental parameters.

2 Preferences and equilibria

In line with preceding studies, I use a reduced-form approach to preferences: I look at conditional-contribution preferences, that is, how much participants are willing to contribute to the public good depending on others' contributions.[4] Fischbacher et al. (2012) show that this approach is behaviourally valid in the sense that contributions in a simultaneous public-good experiment can be explained by participants' elicited conditional-contribution preferences in conjunction with their beliefs.[5] A pure-strategy *revealed-preference Nash-equilibrium* (RPNE) of the simultaneous game then is a contribution profile in which each player chooses a contribution in line with her conditional-contribution preferences given her belief about the other players' contributions, and beliefs match the respective other players' contributions.

To give some examples for RPNE, suppose that two payoff-maximising players are facing a one-shot two-person linear public-good situation. Then, the unique RPNE of the game is the well-known full-defection equilibrium in which no player contributes anything, and this is expected by both players. Suppose now that the two group members have fully altruistic preferences. Then, the unique RPNE would be a full-contribution equilibrium in which both players contribute their full endowment, and either player would expect full contributions by the other.

[3]Following Weibull (2004), I use the term public-good *protocol* to denote a situation in which the *material consequences* have a public-good structure. Whether this translates into a public-good *game* then depends on participants' preferences.

[4]E.g., cf. the references provided in Gächter (2007), Gächter and Herrmann (2009), or Chaudhuri (2011). Conditional-cooperation preferences may be a type of social preferences in their own right, or a manifestation of underlying preferences, e.g., for reciprocity, inequality, or efficiency.

[5]See also my working paper Wolff (2015) that is partially based on the same data as this letter.

Finally, suppose both group members are perfect conditional cooperators.[6] Then, each pure-strategy RPNE is characterised by both contributing some fraction k of their full endowment, $0 \leq k \leq 1$, expecting the other to do the same (which the respective other does).

3 Experimental Design

I use data from three treatments designed to elicit conditional-contribution preferences.[7] Each preference-elicitation treatment consisted of a sequential linear public-good protocol using the strategy method (as in Fischbacher et al., 2001, and—in the three-player 3P.5-treatment—as refined by Cheung, 2013).[8] Table 1 lists the parameters of the three treatments, where n is group size, E is participants' endowment in Euros, m is the public-good multiplier, and the resulting marginal per-capita return is abbreviated to MPCR. Treatment denominations follow the pattern nP(erson)+MPCR: for example, the 3P.5-treatment is a 3-person protocol with an MPCR of 0.5. In all treatments, choices were restricted to six contribution levels. To stick to six levels while keeping profit calculations simple for participants also in the 2P.67-treatment, E had to be adjusted along with m.

Treatment	n	E	m	MPCR
3P.5	3	20	1.5	0.5
2P.75	2	20	1.5	0.75
2P.67	2	15	4/3	2/3

Table 1: Overview of the treatments.

I restricted contributions to six levels because of the importance to elicit the full conditional-contribution vector in the three-player treatment.[9] It is essential to elicit responses to all contribution combinations because the players' response to contributions of, e.g., (8,8) may be very different from their response to (0,16).

[6] Fischbacher et al. (2001) define a *perfect conditional cooperator* to be a player who always wants to match exactly her fellow group members' average contributions.

[7] All three treatments were part of sessions with multiple parts and random rematching between parts. Participants were paid for one randomly-chosen part only, and parts were explained only as soon as they began. There was no feedback on earlier parts before the preference-elicitation treatments, so that I focus on these treatments here. For a detailed discussion of the full experimental setup of what I will call the 3P.5- and the 2P.75-treatments (including the full set of instructions for 3P.5), cf. Wolff (2015). The session setup of the 2P.67-treatment followed a very similar design, the most important difference being that the 2P.67-treatment was the second rather than the fifth part in the session.

[8] In contrast to these papers, I presented first-mover contributions (or contribution combinations) one by one, in an order that was randomised individually for each player.

[9] See Cheung (2013).

For example, a participant with a Fehr-Schmidt utility function would choose 0 in response to (0,16), but 8 in response to (8,8) as long as $\beta > 0.5$.

Prior studies have minimised participants' confusion about the situation by looking at behaviour at the end of repeated-game experiments. I use a different approach, inviting only experienced participants.[10] Table 2 shows an overview of the sessions by treatment. All sessions were conducted at the University of Konstanz' LakeLab between January 2012 and January 2016, using z-Tree (Fischbacher, 2007) and ORSEE (Greiner, 2015). No participant participated more than once.

	3P.5	2P.75	2P.67
Number of sessions	10	3	4
Participants	236	76	82

Table 2: Overview of the sessions by treatment

4 Results

From the literature, we know there is considerable heterogeneity in conditional-contribution preferences. Table 3 shows the distribution of preference-types introduced by Fischbacher et al. (2001; for the classification procedure, see the Online Appendix), alongside the corresponding distribution in each of the treatments of this study. Except for an unusually high fraction of 20% unclassifiables in 2P.67, the distributions are close to what we would expect: because cooperation gets cheaper, a higher MPCR leads to more conditional cooperation and less defection for a fixed group size (2P.67 vs 2P.75), while increasing the group size with (almost) constant multiplier m (2P.75 vs 3P.5 vs Fischbacher et al.'s '4P.4'), has the fraction of conditional cooperators steadily decline as the fraction of defectors increases.[11]

Using the elicited conditional-contribution vectors, I calculate the pure-strategy RPNE sets of all potential matchings within each treatment as detailed in Section 2. I then classify the RPNE set for each of these hypothetical groups according to the cardinality of the RPNE set and according to whether they include full-defection/low-contributions equilibria and high-contributions equilibria. The description of the chosen RPNE set classes and their prevalence in a perfectly-randomised sample are given in Table 4, along with the predicted distribution for the calibrated model of Fehr and Schmidt (1999), as an exemplary benchmark.

[10] Participants in the experiment had participated in at least one public-good experiment and at least four additional other experiments, with no upper limits.

[11] The latter comparison mirrors differences in contribution levels, e.g., between the "LOW_8" and "HIGH_3" treatments in Nosenzo et al. (2015).

Treatment \ Percentage of...	Conditional cooperators	Defectors	Triangle cooperators	Others
3P.5	60	23	11	6
2P.75	76	16	4	4
2P.67	48	21	12	20
Fischbacher et al. (2001); '4P.4'	50	30	14	7

Table 3: Distribution of player types.

Four RPNE-set classes account for 86-93% of all RPNE sets to be expected: (i) a unique, full-defection RPNE, (ii) a unique positive-contributions RPNE (with average contribution levels of 40-45% irrespective of the treatment), (iii) multiple RPNE that range from full-defection to high contributions, (iv) multiple RPNE that include full-defection but no RPNE with average contributions of at least half the endowment. Note that the importance of the type-(ii) RPNE class differs widely between treatments. In particular, it seems to play a substantial role only in the 2P.67-treatment were by some chance, we had an unusually-high percentage of unclassifiable participants. What is important here is that there is a surprisingly high prevalence of multiple-RPNE sets in a well-mixed population for all treatments. For example, the prevalence of multiple-RPNE sets clearly exceeds the predicted frequency on the basis of the Fehr-Schmidt model (see the final row in Table 4). On the other hand, this model does seem to capture the comparative statics between treatments for the two RPNE-set classes that are most prevalent overall, type-(i) and (iii). Note also that—as predicted by all commonly-used social-preference models including Fehr and Schmidt (1999)—the vast majority of all possible matches lead to a RPNE set that includes full-defection.

5 Discussion

In this paper, I documented the distribution of equilibrium-set classes that typical participants would face in the laboratory when presented with linear public-good protocols. The finding that the prevalence of multiple (high-cooperation) equilibria may be higher than commonly expected underlines the necessity of conducting this type of exercise also for other situations in which social preferences are thought to be important. Without studies like the present one, it is impossible to assess how often people face a social dilemma when confronted with a public-good protocol—and how often they face merely a coordination game. Awareness of the prevalence of multiple-equilibrium sets is important because in its light, the ubiquitousness of close-to-omnilateral defection at the end of repeated public-good protocols is surprising and re-opens the question of why the dynamics of

RPNE-set type	Description	3P.5	2P.75	2P.67
$\emptyset$	no pure-strategy RPNE	0.1 (0.0)	1.7 (0.0)	4.2 (0.0)
$\{(0,0,0)\}$	unique RPNE characterised by full defection by all group members	60.1 (93.6)	29.8 (51.0)	38.1 (84.0)
$\{(x,y,z)\}$	unique RPNE where at least one group member's contribution is strictly positive	2.1 (0.0)	11.5 (0.0)	26.4 (0.0)
FULLD-LIMITED	a full-defection RPNE and at least one additional RPNE; the RPNE with the highest average contributions has an average contribution of less than half the endowment	9.6 (0.0)	6.7 (0.0)	5.7 (0.0)
FULLD-INTERMED	a full-defection RPNE and at least one additional RPNE; the RPNE with the highest average contributions has average contributions of between 50% and 80% of the endowment	5.1 (0.0)	3.0 (0.0)	3.4 (0.0)
FULLD-HIGHC	a full-defection RPNE and at least one additional RPNE in which players contribute at least 80% of their full endowment on average	21.4 (6.4)	38.1 (49.0)	17.4 (16.0)
LOWC-HIGHC	a high-contributions RPNE (s.a.) and at least one additional RPNE with average contributions of at most 20% of endowment	0.5 (0.0)	4.9 (0.0)	0.9 (0.0)
ONLYHIGH	at least two RPNE, in all of which average contributions are higher than half the endowment	0.4 (0.0)	0.0 (0.0)	0.7 (0.0)
OTHERS	multiple-RPNE sets that do not fit any of the above categories (88%/51%/41% of these sets include full-defection)	0.7 (0.0)	4.3 (0.0)	3.2 (0.0)
MULTIPLE	cumulated percentage of all multiple-RPNE sets	37.7 (6.4)	57.0 (49.0)	31.3 (16.0)

Table 4: Classification and expected distribution [in percent] of RPNE-set types. Predictions of the calibrated Fehr-Schmidt model are added in parentheses.

play seem to select this equilibrium out of the existing equilibria.

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References

Berg, J., Dickhaut, J., and McCabe, K. (1995). Trust, reciprocity, and social history. Games and Economic Behavior, 10(1):122–142.

Chaudhuri, A. (2011). Sustaining cooperation in laboratory public goods experiments: A selective survey of the literature. Experimental Economics, 14:47–83.

Cheung, S. L. (2013). New insights into conditional cooperation and punishment from a strategy method experiment. Experimental Economics, pages 1–25.

Fehr, E., Kirchsteiger, G., and Riedl, A. (1993). Does fairness prevent market clearing? An experimental investigation. Quarterly Journal of Economics, 108(2):437–.

Fehr, E. and Schmidt, K. M. (1999). A theory of fairness, competition, and cooperation. Quarterly Journal of Economics, 114(3):817–868.

Fischbacher, U. (2007). z-tree: Zurich toolbox for ready-made economic experiments. Experimental Economics, 10(2):171–178.

Fischbacher, U., Gächter, S., and Fehr, E. (2001). Are people conditionally cooperative? Evidence from a public goods experiment. Economics Letters, 71(3):397–404.

Fischbacher, U., Gächter, S., and Quercia, S. (2012). The behavioral validity of the strategy method in public good experiments. Journal of Economic Psychology, 33(4):897 – 913.

Gächter, S. (2007). Conditional cooperation: behavioral regularities from the lab and the field and their policy implications. In Frey, B. S. and Stutzer, A., editors, Psychology and economics, Cambridge, MA. MIT Press.

Gächter, S. and Herrmann, B. (2009). Reciprocity, culture and human cooperation: Previous insights and a new cross-cultural experiment. Philosophical Transactions of the Royal Society B - Biological Sciences, 364(1518):791–806.

Greiner, B. (2015). An online recruitment system for economic experiments. Journal of the Economic Science Association, 1(1):114–125.

Güth, W., Schmittberger, R., and Schwarze, B. (1982). An experimental analysis of ultimatum bargaining. Journal of Economic Behavior and Organization, 3(4):367–388.

Ledyard, J. O. (1995). Public goods: A survey of experimental research. In Kagel, J. and Roth, A., editors, Handbook of Experimental Economics, pages 111–194. Princeton University Press.

Levine, D. K. (1998). Modeling altruism and spitefulness in experiments. Review of Economic Dynamics, 1(3):593–622.

Nosenzo, D., Quercia, S., and Sefton, M. (2015). Cooperation in small groups: the effect of group size. Experimental Economics, 18(1):4–14.

Rabin, M. (1993). Incorporating fairness into game theory and economics. The American Economic Review, 83(5):1281–1302.

Weibull, J. W. (2004). Testing game theory. In Huck, S., editor, Advances in Understanding Strategic Behaviour: Game Theory, Experiments and Bounded Rationality. Essays in Honour of Werner Güth, pages 85–104. Palgrave Macmillan.

Wolff, I. (2015). When best-replies are not in equilibrium: Understanding cooperative behaviour. Research Paper Series 97, Thurgau Institute of Economics.

Online Appendix Player-type classification

To give an overview of the player types, I characterise them along the lines of Fischbacher, Gächter, and Fehr (2001). However, given the more precise measurement of preferences in the 3P.5-treatment, some adjustments are needed.[12] To account for the modified setup in the three-player treatment, I group the other-player contribution combinations into three sets of seven combinations [two contributions, for the 2P.75- and the 2P.67-treatments] each, using the following characterisations:[13]

Conditional cooperators. Participants were categorised as conditional cooperators if the following conditions held simultaneously: their conditional contributions for intermediate (high) other-player contributions were at least as high as for low (intermediate) contributions, the difference between conditional contributions for high and low other-player contributions was at least $\frac{20}{7}$ [$\frac{20}{2}$, $\frac{15}{2}$, for 2P.75 and 2P.67],

[12]For consistency, I followed a similar procedure in the two-player treatments, averaging responses over pairs of other-player contributions.

[13]For this grouping, I ordered the other-player contribution combinations by the respective empirical average response to them; using the combinations' means and variances lexicographically yields the same sets. The conditional contributions were averaged within the subsets in order to allow for minor inconsistencies that may arise due to the random-order one-by-one presentation of the possible contribution combinations of the other players.

and their response to others' full contribution was not 0.[14] In 2P.67, I manually classified four additional participants as conditional cooperators, three of them having a Spearman correlation coefficient of contributions and responses with a one-sided significance level of less than 5%. The fourth participant responded to (0,3,6,9,12,15) by (0,3,6,9,0,15), which I interpreted as perfect conditional cooperation with a single error.

Defectors. Participants were categorised as defectors if their average conditional contributions did not surpass a value of 2 for low, intermediate, and high other-player contributions.

Triangle contributors. Participants were categorised as triangle contributors if their average conditional contributions were strictly higher for intermediate other-player contributions than for low or high ones, or if their average conditional contributions increased monotonically in the other-player contributions but they would respond to others' full contributions by defecting. In 2P.67, I manually classified three additional participants as triangle contributors, who had monotonically-increasing response vectors with a downward kink only at the first-mover's full-contribution.

Others. Participants were categorised as 'others' if they would not fit into any of the above three categories.

[14] The difference of $\frac{20}{7}[\frac{20}{2}, \frac{15}{2}]$ was chosen to include players who would choose $20[20, 15]$ in response to the full-contribution combination 20–20[contributions of $20, 15$], and 0 for all other contribution combinations. This was the case for seven (3%) 3P.5-participants.

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