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Abstract: The paper discusses the impact of performance based selection in secondary education on student incentives. The theoretical approach combines human capital theory with signaling theory. The consideration of signaling offers an explanation for observed performance of educational systems with a standard peer effect argument. More specifically it can be optimal to select students according to ability even if selective systems do not outperform comprehensive systems in tests. Selection achieves the same output with lower private costs for the students. The paper questions the strong focus on educational tests to measure the efficiency of selective systems as long as these tests provide no information about a student's incentives and private costs.

Keywords: Education, signalling, selection, ability grouping, incentives

JEL Classification: I 20, I 28

1 Introduction

Selection is a key management instrument in educational organizations. Theoretical models often imply that ability grouping (or tracking or selection) is efficient. Assumptions about peer effects are one reason for this result. Peers enhance the marginal productivity of a student and this benefit typically increases in the ability of the peers. If this peer effect and individual ability of a student are complementary inputs then high ability students should be matched with other higher ability students into specific learning groups. Yet robust empirical foundations for this sort of peer interaction and their implications are hard to come by. In particular selective systems do not outperform comprehensive ones in international comparison tests like the PISA-Study (e.g. Hanushek/Woessmann 2006).

This apparent contradiction either implies that the standard assumptions about peer interaction are wrong or that a more specific theoretical explanation is required. Such an explanation should eliminate the contradictions between theory and empirical result. This paper focuses on the latter and provides three contributions towards the understanding of selection in educational systems.

1. The paper takes signaling activities into account. Selection has an important impact on the value of education because the placement of a student into a specific school or class provides additional information for a less informed party about the 'raw' ability of a student.
2. The consideration of signaling offers an explanation for observed performance of educational systems with a standard peer effect argument. More specifically it can be optimal to select students according to ability even if selective systems do not outperform comprehensive systems. Selection achieves the same output with lower private costs for the students.
3. The paper questions the strong focus on educational tests to measure the efficiency of selective systems as long as these tests provide no information about a student's incentives and private costs.

Educational tests are certainly informative but the results show that they require a careful interpretation. Selection influences incentives which again have an impact on educational test scores. Ignoring these incentive effects can imply inaccurate estimations regarding the outcomes of selection as a policy instrument. This paper therefore provides a theoretical contribution to a largely empirical literature. I do not argue against evidence-based policy making. However, a purely data-driven evaluation can lead to misleading results if important aspects are beyond observation.

The argument of the paper can be summarized as follows. The analysis focuses on student effort provision. Students invest effort into their education for two reasons. Firstly, effort augments their skills and more skills translate into higher salaries after education. Secondly, schooling is a signal about a student's 'raw' or innate ability which is unobservable. Students with higher ability have, on average, better marks. Innate ability directly affects productivity after education. It helps acquiring job-specific human capital or provides general problem-solving skills. Hence, employers want to identify ability and adjust wage offers accordingly. School performance provides valuable information about a student's ability. This signaling aspect induces students to provide even more effort.

Educational policy shape effort provision. The incentives for a student differ between a selective and a comprehensive educational system. In most selective systems, primary education is comprehensive and secondary education is selective. Some schools enroll the high ability students the other schools teach students with lower abilities. If peer effects and signaling are important then students want to get to school with the high-ability students. Therefore, the students in a selective system work harder in primary education. These students will outperform students from comprehensive systems in standardized tests even if they study in otherwise identical primary schools.

In secondary education the incentives turn around. Ceteris paribus, students in comprehensive systems will provide more effort. For them, the final performance in school is the only clearly observable ability signal. Students in a selective system will work less hard because selection already has provided some relevant information about their ability. This difference can explain why potential gains in peer group effects from selection are so difficult to observe and why selective systems do not produce higher test scores. Note however, that the variance in effort provision can be much larger among students in a selective system. The positive impact of good fellow students and the resulting increase in marginal productivity compensate reduction in incentives. In the lower track the rather poor peer effects increase the disparity in incentives.

Therefore, a differences-in-differences methodology like in Hanushek and Woessmann (2006) or a similar approach discussed in Pischke and Manning (2006) underestimate the impact of selection on educational performance. Section E is dedicated to this measurement problem.

The discussion about optimal education policy has largely ignored incentives for students, although empirical studies show that incentives matter (De Fraja et al 2005, Kremer et al. 2004 with students from Kenya, Angrist and Lavy 2003 with Israeli student data). De Fraja and Landeras (2006) provide an inspiring theoretical contribution to the research of incentives in education. They show that increasing teacher incentives via more competition among schools may backfire on student incentives in a similar way.

Furthermore, few studies on education policy consider the well established literature about education as a signal in the labor market (starting with Spence 1973). Bedard (2001) shows how dropout decisions in American high school confirm signaling arguments but contradict standard human capital theory. Unlike Bedard (2001) this paper investigates incentives when schooling is compulsory and selection is based on performance. De Fraja and Landeras (2006), Brunello and Giannini (2005) and Lee (2007) provide further exceptions. Brunello and Giannini (2005) also show why neither perfect sorting nor comprehensive schooling provides a strictly dominant solution, although with an entirely different treatment of the signalling problem. For them (Brunello/Giannini, 2005, p. 190)

"[s]tratified systems trade the advantages of specialization and signalling against the disadvantages of producing skills with limited flexibility and versatility."

In Brunello and Giannini (2005), the students cannot influence the admission result and the subsequent signal with more effort. "Their" students have technical and academic abilities and schools provide different types of education. In contrast to Brunello and Giannini (2005), this paper provides a policy instrument to manipulate the sorting process. I do not discuss when students should optimally be separated. This problem is analyzed in Brunello et al. (2004). Lee (2007) shows how national differences in the optimal timing of signals shape student behavior.

Many contributions to the economics of education make assumptions about how students influence their fellow students. Some authors (e.g. Epple et al. 2003), suggest that students benefit from the ability of their fellow students. Hence, if the average student in a class is a rather able one, the learning conditions for each student in this class are rather good. Able students may provide more help or induce greater interest in academic problems. Other authors emphasize homogeneity. If ability varies considerably among students in a class it creates a negative learning environment (e.g. Dobbelsteen et al. (2002)). One motive behind this idea is the willingness to cooperate. Students are more likely to interact if they face the same problems. Lazear (2001) discusses possible negative externalities of student behavior. Students can interrupt education and distract fellow students. All these approaches imply that *ceteris paribus* the (self)selection of students in homogeneous groups -with respect to ability or the propensity to interrupt- is efficient. “Good” students provide more positive external effects. They also get higher benefit from these effects because of higher marginal productivity.

Econometric problems restrict the identification peer effects. The problem in many empirical studies (e.g. Hoxby 2000, 2001, McEwan 2003 , Hanushek et al. 2003, Cullen et al. 2003, Ammermüller/Pischke 2006) is the identification of exogenous ability measures to get around the so called "reflection problem" (see Manski 1993). Then, the (self) selection of students into different peer groups restricts the identification of what would have happened to the same student in another group. (Quasi-)Experimental studies may bring more insight but they are still in short supply. Sacerdote (2001) provides evidence for peer effects in dorms at Dartmouth College, where assignment to the rooms is a random process. A learning experiment with Swiss students shows that good students benefit from other students but this benefit is independent of the ability of the learning partners (Eisenkopf 2007). If this result can be confirmed in further experiments, it contradicts the standard peer effect assumptions. Finally, other reasons exist why a *sui generis* positive peer group effect can produce negative external effects on the outcome from ability grouping, e.g. higher failure rates for high-ability students (see Meier 2004).

To support the argument, section B introduces the analytic framework. Section C identifies the resulting first-best solution. In section D intertemporal effects are analyzed. The section shows that selection in secondary education does not entirely reduce signalling incentives but shifts them to primary education. Section E discusses the econometric implication of this finding with a particular reference to Hanushek and Woessmann (2006). Section F derives empirically testable hypotheses from the model. It concludes with a discussion of policy implications.

2 The model

Suppose the existence of one region with two schools $k \in \{a,b\}$ of equal size. Each school must enroll half of the students in the region. The mass of students in the region is normalized to one. Students are heterogeneous with respect to their ability θ. The ability is distributed symmetrically around $\hat{\theta}$ according to the differentiable distribution function $F(\theta)$, with $F'(\theta) = f(\theta)$. The ability of student *i* is known by the student, but not by the future employer of the student. Employers know the distribution of ability among the students. Students can invest in effort *e* to improve their educational qualification. The costs of effort follow the function $C(e)$, with $C'(e) > 0$ and $C''(e) > 0$. Only the student knows about his choice of the effort level.

Policy makers can choose between a comprehensive and a selective educational system, i.e. between sorting and integrating students. Under a sorting (or tracking) policy, high ability students go to one school and low ability students to the other one. Otherwise, the ability distribution is identical in both schools. For the moment it is assumed that sorting is perfect. The choice between a selective and a comprehensive system has an impact on average ability $\bar{\theta}_k$ in a school (the peer effect). Thus the contributions to educational output are defined: effort, ability and the peer effect.

Assumption 1 *A student's educational output or qualification is* $q(e,\theta;\bar{\theta}_k)$, *and satisfies* $\frac{\partial q}{\partial e}, \frac{\partial q}{\partial \theta}, \frac{\partial q}{\partial \bar{\theta}_k} > 0$ *as well as* $\frac{\partial^2 q}{\partial e^2}, \frac{\partial^2 q}{\partial \theta^2} < 0$. *All cross derivatives are strictly positive.*

Unlike in De Fraja and Landeras (2006) the observation of the qualification is distorted, i.e. the marks do not accurately reflect the actual skills.

Assumption 2 *A student's* observed *educational output or qualification is*

$$Q = q + \epsilon \qquad (1)$$

The random variable ε is iid across students and follows the distribution G(ε) with the density function g(ε). The expected value of ε is zero and g(ε) is positive for all ε. The distribution is single-peaked and symmetric around the expected value.

Imperfect observation makes education risky. However, the paper does not discuss the implications of risk aversion but the impact of this imperfection on risk neutral students. Therefore, students and employers are risk-neutral. After education, a student enters the labor market.

Assumption 3 *A student's labor market output depends on his educational qualification (q) and his ability (θ). The labor market output is Π(θ,q(θ)), with* $\frac{\partial \Pi}{\partial \theta}, \frac{\partial \Pi}{\partial q} > 0, \frac{\partial^2 \Pi}{\partial \theta^2}, \frac{\partial^2 \Pi}{\partial q^2} < 0, \frac{\partial^2 \Pi}{\partial \theta \partial q} \geq 0$

Assume further that $\frac{\partial^2 \Pi}{\partial \theta^2}$ *is in a parameter range such that the optimal effort supply is always positive but not infinite.*

Since employers cannot observe the ability θ they have to estimate θ and subsequently the expected labor market output from the observable qualification. The term $\mu(\theta;Q)$ denotes the density of an employer's belief about the ability of an individual whose observed qualification is Q. Belief formation takes place after the qualifications have been published. In a competitive labor market initial wage offers for a student with observed qualification Q are equal to expected productivity $B(Q)$.

$$w = B(Q) = \int_\theta \int_\epsilon \Pi(\theta, Q-\epsilon)\mu(\theta; Q-\epsilon)g(\epsilon)d\theta d\epsilon \quad (2)$$

Note that the expected wage offer for distorted qualification signals is lower than the offer for accurate ones. Labor market output $\Pi(\theta,q)$ is concave in both arguments and the actual qualification q again is a concave function of θ. The random variable ε, which distorts observed qualification Q, is symmetrically distributed around its expected value. Due to the concavity of the labor market output the employer's losses from $q<Q$ are greater than the gains from $q>Q$. Hence the employer has to offer a lower wage than in the case of perfectly observable qualifications. Otherwise the losses of students with a high ability from relatively poor observable qualifications ($Q<q$) would be smaller than the gains of less able ones from relatively good observable qualifications ($Q>q$).

If the employer can learn the actual ability some time after employment has started, any new wage offer reflects the actual output $\Pi(\theta,q)$. For simplicity reasons, such a possibility is not considered in the analysis. It is discussed in De Fraja and Landeras (2006) and has no qualitative impact on the results

3 To sort or not to sort?

The analysis starts with the comparison of the two extreme cases. In the comprehensive case (no sorting) students are randomly assigned to the two schools. In the (perfect) sorting case, the more able students go to one school, the less able to the other one. A perfect selection test does exist. I will discuss imperfect selection later in the paper. As in De Fraja and Landeras (2006), the analysis is restricted to a pure strategy equilibrium with the following characteristics. Student effort does not decrease in ability. The employer formulates beliefs about a student's ability such that the resulting wage offers increase in observed performance.

The analysis focuses on the effort provision of a student who is motivated by his prospective wage offer. This wage offer is affected by the school performance which influences productivity and the employers' beliefs. School performance depends on the selection policy. Therefore I compare effort provision in a sorting and a comprehensive regime.

3.1 No sorting

The ability composition in both schools is identical, which is denoted by the superscript *ns*. The respective peer effect is given by $\overline{\theta}_k^{ns}$, such that $\overline{\theta}_a^{ns} = \overline{\theta}_b^{ns} = \hat{\theta}$. The objective of a student is to maximize the difference between the expected wage and his costs of effort by his choice of effort. The student correctly anticipates the wage formation of future employers and the effort supply of his fellow students. As stated above the social externality of his qualification is irrelevant to any student. The maximization problem is

$$\max_e B(Q^{ns}) - C(e) \qquad (3)$$

with

$$B(Q^{ns}) = \int_\theta \int_\epsilon \Pi(\theta, Q^{ns} - \epsilon)\mu(\theta; Q^{ns} - \epsilon)g(\epsilon)d\theta d\epsilon$$

The transformation of the first order condition yields

$$\left(\frac{\partial B(Q^{ns})}{\partial Q^{ns}} + \frac{\partial B(Q^{ns})}{\partial \int_\theta \mu(\theta; Q^{ns})d\theta} \frac{\partial \int_\theta \mu(\theta; Q^{ns})d\theta}{\partial Q^{ns}} \right) \frac{\partial Q^{ns}}{\partial e} = C'(e) \qquad (4)$$

The first summand shows the marginal increase in productivity, the second one the marginal change in the employers' beliefs. Better educational results increase the employers' expectation about the student's ability. However, all fellow students provide additional effort as well and the expected labor market output per student still increases strictly in ability.

3.2 Sorting

In a selective schooling system, the students with above-average ability $(\theta > \hat{\theta})$ go to school *a*, the others go to school *b*. Assume for simplicity that a costless test is available to identify perfectly if a student is above or below this threshold. Of course, after sorting peer effects at schools *a* and *b* have the following property: $\overline{\theta}_a^s > \overline{\theta}_b^s$. The superscript *s* indicates that the students have been selected. Employers know that the ability of a student at a school *a* cannot be below $\hat{\theta}$ and adjust their beliefs accordingly. Now, the term $\mu(\theta; Q^s \mid \theta > \hat{\theta})$ denotes the density of an employer's belief about the ability of a student at school *a* whose observed qualification is Q^s. The density function is truncated at $\hat{\theta}$. For students at school *a*, the truncation is at the lower end. For students at school *b*, the truncation is at the upper end.

The maximization of a student at school *a* changes into:

$$\max_e \int_{\hat{\theta}} \int_\epsilon \Pi(\theta, Q^s)\mu(\theta; Q^s \mid \theta > \hat{\theta})g(\epsilon)d\theta d\epsilon - C(e) \qquad (5)$$

Let $B_a(Q^s,\theta>\hat{\theta})$ denote $\int_{\hat{\theta}}\int_{\epsilon}\Pi(\theta,Q^s)\mu(\theta;Q^s,\theta>\hat{\theta})g(\epsilon)d\theta dq^s$. The first order condition turns into

$$\left(\frac{\partial B_a(Q^s,\theta>\hat{\theta})}{\partial Q^s}+\frac{\partial B_a^s(Q^s,\theta>\hat{\theta})}{\partial\int_{\hat{\theta}}^{\infty}\mu(\theta;Q^s\,|\theta>\hat{\theta})d\theta}\frac{\partial\int_{\hat{\theta}}^{\infty}\mu(\theta;Q^s\,|\theta>\hat{\theta})d\theta}{\partial Q^s}\right)\frac{\partial Q^s}{\partial e}=C^{'}(e) \qquad (6)$$

For a student at school *b*, optimal effort is like-wise

$$\left(\frac{\partial B_b^s(Q^s\,|\theta<\hat{\theta})}{\partial Q^s}+\frac{\partial B_b^s(Q^s\,|\theta<\hat{\theta})}{\partial\int_{-\infty}^{\hat{\theta}}\mu(\theta;Q^s\,|\theta<\hat{\theta})d\theta}\frac{\partial\int_{-\infty}^{\hat{\theta}}\mu(\theta;Q^s\,|\theta<\hat{\theta})d\theta}{\partial Q^s}\right)\frac{\partial Q^s}{\partial e}=C^{'}(e) \qquad (7)$$

If a policy maker maximizes the aggregate utility of all students the following result can be obtained.

Proposition 1 *A perfect selection is the dominant policy even if comprehensive education leads to higher observed qualifications.*

Students at school *a* in the sorting regime face a better peer group than they would in a comprehensive schools. Students at school *b* face a worse peer group. Given the complementarities in the production function the increase in qualification for students at school *a* is greater than the loss for students at school *b*. Students in comprehensive schools have to provide more effort to get the same qualification. This logic is applicable to a comparison between a perfect selection and any imperfect one. Appendix A provides a more specific proof of the proposition

Proposition 1 holds even if comprehensive education leads to higher qualifications. In a comprehensive school signaling is a relatively strong motive for effort provision. In this school student effort influences the employers' belief formation more strongly than in a selective one. This difference is particularly relevant for students with an average ability. With perfect selection students at school *b* with good observed qualifications will never be as able as students with a poor record at school *a*. Hence, comprehensive schools may elicit more effort and higher qualifications than schools in a selective system even though it is less productive.

Higher qualifications from higher effort levels may increase the wage offers for a student such that $\int_\theta B_k^{ns}(\theta) f(\theta) d\theta > \int_\theta B_k^{s}(\theta) f(\theta) d\theta$ holds. Yet, the additional costs of effort outweigh this increase in wage offers. Otherwise, equations (6) and (7) would be violated.

Therefore higher qualifications are a result of an overinvestment. Comprehensive education can be the optimal solution, if qualification or effort provision generates positive external effects, e.g. happy parents.

The properties of the peer effect also show which students prefer which educational system.

Corollary 1 *Students with $\theta > \hat{\theta}$ prefer a selective schooling system, students with $\theta < \hat{\theta}$ prefer a comprehensive one. Sorting increases the inequality in educational qualification between these two types of students.*

The expected marginal productivity is greater for high ability students ($\theta > \hat{\theta}$) in the selective system and greater for low ability students ($\theta < \hat{\theta}$) in the comprehensive system. The inequality in qualification increases in a selective system as good students gain from improved peer effects and low ability students lose accordingly.

Students with a high expected qualification will leave a comprehensive school system, if they can also go to a selective one, e.g. a private school or a selective school in a neighboring jurisdiction. Of course, such an outside option undermines potential benefits from comprehensive schooling.

4 Intertemporal incentives

Almost all educational systems do not sort in elementary school but at some later point. In this section, the incentives from a selective school system for pre-selection students are analyzed. The results from the analysis have implications for econometric research which will be discussed in a subsection.

Consider a two period process. In period t=1 (or in primary education) all students are educated in identical schools. In period t=2 (or secondary education) the students are either sorted or they are still kept in identical schools. In the selective case, sorting depends on the observed output in the first period (Q_1). This criterion allows for a cheap and simple selection mechanism. I assume that alternative selection instruments (tests, interviews etcs.) are prohibitively expensive. If Q_1 is greater than $\hat{q}$, the student will be sorted into better peer group. The term $\hat{q}$ denotes the expected output of a student with median ability in equilibrium. Of course, this is an endogenous variable. Given an infinite number of students and effort provision increasing in ability this expected output constitutes a quasi-exogenous threshold value.

Students can choose their respective effort level at the start of each period. The qualification in period 2 additionally depends on the actual qualification after period 1, such that

$$Q_2 = q(e_2, \theta, q_1; \overline{\theta}_k)) + \varepsilon_2 \tag{8}$$

The separation into two periods changes the incentive structure of education as the majority of students in a tracking system have higher incentives in the first period. Selection generates incentives because higher effort increases the chance to be promoted to the good school with a better peer group. In particular, high ability students are motivated by the improved learning environment. For students with average ability, the gap between the two prospective peer groups induces them to work hard. On the other hand low ability students know that they are very likely to face a rather poor peer group and will provide rather low effort. Proposition 2 identifies the effort supply of students in period 1 depending on the educational policy and individual ability under *ceteris paribus* conditions. A comparison of effort supply functions in period 1 in the different regimes yields these results.

Proposition 2 *Assume that - for a given qualification* q_1 *from period 1 - the expected additional qualification in secondary education (Δ(s,ns)) is identical in the sorting and in the non-sorting regime.*

$$\Delta(s, ns) = \int_\theta Q_2^s(\theta) - Q_1^s(\theta) d\theta - \int_\theta Q_2^{ns}(\theta) - Q_1^{ns}(\theta) d\theta = 0 \tag{9}$$

Then, the following results hold:

1. *First period qualification in the sorting regime is lower for the students with sufficiently low ability (i.e.:* $\lim_{\theta \to -\infty} \left(q(\theta)_1^s - q(\theta)_1^{ns} \right) < 0$ *) and higher for the median student and for students with very high ability.* $\left(q(\theta \geq \hat{\theta})_1^s > q(\theta \geq \hat{\theta})_1^{ns} \right)$

2. *Increasing uncertainty (increasing* σ^2*) reduces the incentives for the students with median ability and increases the incentives for the students at the margins of the ability distribution.*

Proof. See Appendix B

Proposition 2 shows that selection increases the average student input even if selective secondary education does not imply a higher productivity. These increased differences in inputs lead to even greater inequalities in educational output at the end of the education. Result 2 is similar to results from bonus or tournament models. Increased uncertainty increases the likelihood of passing (or failing) for low (high) ability students. Hence, these students work harder to achieve the desired result. For the median student, the impact of effort on the outcome decreases, hence the incentives are lower.

5 Econometric Implications

The implication of proposition 2 is crucial for the empirical literature. The evaluation of tracking policies (e.g. Argys et al. 1996; Betts/Skolnick 2000; Figlio/Page 2002) often suffers from a lack of independent observations. It is also difficult because unmeasured factors can bias the estimation results, e.g. the impact of other policies or differences in teacher quality. The differences-in-differences approach by Hanushek and Woessmann (2006) controls for these institutional differences across countries, as long as tracking and non-tracking countries do not differ systematically in other relevant policies. The approach and the available data set of Hanushek and Woessmann do provide a valuable insight into the impact of tracking on the distribution of educational outcomes. Yet, their methodological improvement in one area raises another objection with respect to overall outcome. Hanushek and Woessmann (2006, p. 66) claim that

> "[t]he impact of tracking can (...) be estimated by comparing the average achievement gain in tracked countries to that in untracked countries."

Such a differences-in-differences methodology tests if equation (10) in proposition 2 is larger, smaller or equal to zero. However, due to result 1 in the previous proposition, $\Delta(s,ns)$ does not capture that tracking ceteris paribus leads to higher performance in primary education $\left(\int_{\theta} Q_1^{s}(\theta)d\theta > \int_{\theta} Q_1^{ns}(\theta)d\theta\right)$. Equation (10) just captures the impact of tracking on the obtained qualifications in *secondary* education. The full impact of tracking is underestimated. Note that recent empirical contributions have questioned the suitability of the approach and the robustness of its results (Waldinger, 2007, Pischke/Manning, 2006).

6 Conclusion

The paper has discussed a reason why positive effects from the selection of students are not observable, even if standard assumptions about peer group effects are not rejected. Sorting already provides a signal about unobservable ability. Hence, student performance in selective secondary education has less impact on the beliefs of an employer than in a comprehensive system. Introducing ability grouping transfers incentives from secondary into primary education. If the assumptions about peer effects are correct, then sorting is the dominant policy. Equality in test scores with identical financial inputs does not reflect unobservable effort provision. However, the case for further research on the properties of peer effects is obvious.

The results allow deriving empirically testable hypotheses. Firstly, performance in primary education is higher in systems where selection takes place in secondary education. Primary education has a higher impact on labor market success in selective systems. Hence, differences-in-differences estimations do not capture the full impact of tracking.

Secondly, proposition 1 implies that comprehensive education requires higher effort contributions from students to acquire the same qualification. If this hypotheses can be confirmed it questions a popular assumption that improving qualification with given financial resources also improves welfare. This assumption only holds if the social benefits from education are sufficiently large.

Beyond the empirical hypotheses the paper contributes to the understanding of educational testing and policy evaluation. The results seemingly imply that selection according to ability is an efficient educational policy. More precisely however, they state that the evidence is not as contrary to that statement as recent studies suggest. The results support the hypothesis that selection increases the inequality in educational outcome.

Perhaps the most important contribution of the paper is towards the understanding of educational test results. Standardized international performance tests are a powerful tool to evaluate both schools and policies. For good reasons they are widely used in the social sciences and they become an increasingly popular evaluation tool for policy makers. However, the analysis in this paper has shown that an analysis of test scores should account for underlying incentives. It is the investigation of incentives which distinguishs economics from other social sciences. Hence, it is the topic where economists can provide the greatest impact for educational research.

Appendix A: Proof of Proposition 1

Proof. The marginal increase in observed qualification is greater if students are tracked according to their ability.

$$\int_{\theta} Q_{e}^{s} f(\theta) d\theta > \int_{\theta} Q_{e}^{ns} f(\theta) d\theta \qquad (10)$$

with $Q_e^s = \dfrac{\partial Q(q(e(\theta),\theta,\overline{\theta}_k^s))}{\partial e}$ and $Q_e^{ns} = \dfrac{\partial Q(q(e(\theta),\theta,\overline{\theta}_k^{ns}))}{\partial e}$

Students at school *a* in the sorting regime face a better peer group than they would in a comprehensive schools. Students at school *b* face a worse peer group. Ability is symmetrically distributed around $\hat{\theta}$, such that average ability in both schools is equidistant from the average ability in the entire population $(\hat{\theta}-\overline{\theta}_b=\overline{\theta}_a-\hat{\theta})$ and students at the better school have a greater marginal productivity ($\frac{\partial^2 q}{\partial\overline{\theta}_k\partial\theta}>0$, see assumption 2). Hence, the increase in qualification for students at school *a* is greater than the loss for students at school *b*. Students in comprehensive schools have to provide more effort to get the same qualification. The logic is applicable to a comparison between a perfect selection and any imperfect one. ■

Appendix B: Proof of Proposition 2

The analysis of intertemporal incentives is backwards. Period 2 is analogous to what has been discussed in the previous section, with two notable exceptions. Firstly, the peer group effects are different because sorting is based on the observable output Q_1. which depends on a random variable. Hence, some less able or less engaged students will slip into the better peer group and vice versa. This exception does not imply changes in the qualitative results of the previous section. Secondly, the impact of the output in period 1 has to be taken into account. Therefore, further qualitative differences between a selective and comprehensive system stem from differences in the pre-selection period.

In this period 1, consider first the non-sorting case. The problem of a student is the following:

$$\max_{e_1} B^{ns}(Q_2(q_1))-C(e_1) \tag{11}$$

which implies

$$\left(\frac{\partial B^{ns}(Q_2)}{\partial Q_2}+\frac{\partial B^{ns}(Q_2)}{\partial\int_{\theta}\mu(\theta;Q_2)d\theta}\frac{\partial\int_{\theta}\mu(\theta;Q_2)d\theta}{\partial Q_2}\right)\frac{\partial Q_2}{\partial Q_1}\frac{\partial Q_1}{\partial e_1}=C'(e) \tag{12}$$

The school indicator *k* has been ignored in the notation because all schools are identical in both periods. In the comprehensive school system students in elementary school are only motivated by the effect on their post-educational wages.

For the sorting case, a student has to solve

$$\max_{e_1} \Pr\left(Q_1 > \hat{q}\right) B^s(Q_2(q_1) \mid Q_1 > \hat{q}) + \left(1 - \Pr\left(Q_1 > \hat{q}\right)\right) B^s(Q_2(q_1) \mid Q_1 < \hat{q}) - C(e_{i,1}) \qquad (13)$$

From the relevant first order condition follows

$$\begin{aligned} & g\left(Q_1 - \hat{q}\right) \Delta B + \Pr\left(Q_1 > \hat{q}\right) \frac{\partial B^s(Q_2(q_1) \mid Q_1 > \hat{q})}{\partial q_1} \frac{\partial q_1}{\partial e_1} \\ & + \left(1 - \Pr\left(Q_1 > \hat{q}\right)\right) \frac{\partial B^s(Q_2(q_1) \mid Q_1 < \hat{q})}{\partial q_1} \frac{\partial q_1}{\partial e_1} = C'(e_{i,1}) \end{aligned} \qquad (14)$$

Notation in the equation is simplified, with

$$\Delta B = B^s(Q_2(q_1) \mid Q_1 > \hat{q}) - B^s(Q_2(q_1) \mid Q_1 < \hat{q}),$$

$$\frac{\partial B^s(Q_2(q_1) \mid Q_1 > \hat{q})}{\partial q_1} =$$

$$\left(\frac{\partial B^s(Q_2(q_1) \mid Q_1 > \hat{Q})}{\partial Q_2} + \frac{\partial B^s(Q_2(q_1) \mid Q_1 > \hat{Q})}{\partial \int_\theta \mu(\theta; Q_2, Q_1 > \hat{Q}) d\theta} \frac{\partial \int_\theta \mu(\theta; Q_2, Q_1 > \hat{Q}) d\theta}{\partial Q_2} \right) \frac{\partial Q_2}{\partial q_1}$$

and

$$\frac{\partial B^s(Q_2(q_1) \mid Q_1 < \hat{q})}{\partial q_1} =$$

$$\left(\frac{\partial B^s(Q_2(q_1) \mid Q_1 < \hat{Q})}{\partial Q_2} + \frac{\partial B^s(Q_2(q_1) \mid Q_1 < \hat{Q})}{\partial \int_\theta \mu(\theta; Q_2, Q_1 < \hat{Q}) d\theta} \frac{\partial \int_\theta \mu(\theta; Q_2, Q_1 < \hat{Q}) d\theta}{\partial Q_2} \right) \frac{\partial Q_2}{\partial q_1}$$

Recall that *g*(.) is the density function of the random variable. The term $g\left(Q_1 - \hat{q}\right) \Delta B$ in equation (14) denotes the increase in probability to get the higher returns from better schooling. Due to the properties of the error term, the following conditions hold for students with very high expected output and for students with very low expected output:

$$\lim_{q\to-\infty} g\left(Q_1-\hat{q}\right)=\lim_{q\to\infty} g\left(Q_1-\hat{q}\right)=0 \tag{15}$$

$$\lim_{q\to-\infty} \Pr\left(Q_1>\hat{q}\right)=1-\lim_{q\to\infty} \Pr\left(Q_1>\hat{q}\right)=0 \tag{16}$$

Very able students are almost certain to go to school *a*. Students with very low ability are bound for school *b*. Additional effort basically does not affect the selection outcome.

The peer group effects imply that very good students benefit from increased marginal productivity in a selective system, while low ability students face a loss in productivity (see proof of proposition 1):

$$\frac{B^s(Q_2(q_1)\,|Q_1>\hat{q})}{\partial q_1}\frac{\partial q_1}{\partial e_1}>\frac{B^{ns}(Q_2,q_1)}{\partial q_1}\frac{\partial q_1}{\partial e_1}$$

and

$$\frac{B^s(Q_2(q_1)\,|Q_1<\hat{q})}{\partial q_1}\frac{\partial q_1}{\partial e_1}<\frac{B^{ns}(Q_2(q_1))}{\partial q_1}\frac{\partial q_1}{\partial e_1}$$

These implications establish the results for the high ability students and the low ability students.

For the student with $\theta=\hat{\theta}$,

$$g\left(Q_1-\hat{q}\right)\Delta B+\Pr\left(Q_1>\hat{q}\right)\frac{\partial B^s(Q_2(q_1)\,|Q_1>\hat{q})}{\partial e_1}+\left(1-\Pr\left(Q_1>\hat{q}\right)\right)\frac{\partial B^s(Q_2(q_1)\,|Q_1<\hat{q})}{\partial e_1} > \frac{\partial B^{ns}(Q_2(q_1))}{\partial e_1} \tag{17}$$

has to hold in order to satisfy the proposition. Since both schools have the same amount of students, the expected qualification in period 1 for a student is defined by the threshold value $\hat{q}$ such that $g\left(E(Q_1)-\hat{q}\right)=g(0)>0$. The peer effects imply

$$\Pr\left(Q_1>\hat{q}\right)\frac{\partial B^s(Q_2(q_1)\,|Q_1>\hat{q})}{\partial e_1}+\left(1-\Pr\left(Q_1>\hat{q}\right)\right)\frac{\partial B^s(Q_2(q_1)\,|Q_1<\hat{q})}{\partial e_1}\geq\frac{\partial B^{ns}(Q_2)}{\partial e_1} \tag{18}$$

which establishes equation (17).

For result 2 notice that $g\left(Q_1 - \hat{q}\right)$ increases with increasing uncertainty at the margins of the distributions of possible output and decreases at $E(Q_1) = \hat{q}$. ■

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