No. 30 june 2008

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Research Paper Series
Thurgau Institute of Economics and Department of Economics
at the University of Konstanz

THURGAU INSTITUTE
OF ECONOMICS
at the University of Konstanz

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This version: June 2008

Abstract

This paper develops an endogenous timing model for a quantity-setting duopoly with imperfect information on market demand and costly market research. If the market research cost K is too high, market research never plays a role. For intermediate values of K, and independently of production costs, there are two SPNE with endogenous leadership. If K is low, SPNE with endogenous leadership appear if the production costs of the leader are low enough relative to market conditions (e.g. large expected market capacity and small variance thereof). If both firms are relatively inefficient, there is a SPNE with simultaneous production.

JEL classification: L13; D21; D82

Keywords: Endogenous timing; Market research; Endogenous leadership.

1 Introduction

In classical duopoly models, the timing of firms' choices (e.g. production) is exogenously given. For instance, firms are assumed to act simultaneously in Cournot and Bertrand duopolies, and one firm is arbitrarily chosen to take action first in the Stackelberg game or the price leadership model. Of course, the industrial organization literature has long questioned whether and when such settings are realistic, giving rise to a large number of studies which aim to explain the firms' timing choices endogenously.

A seminal contribution in this area comes from Hamilton and Slutsky (1990) (hereafter HS). They develop two different two-period duopoly models. The first is a game of timing with observable delay, which requires each firm to announce its timing choice first and then to commit to it. For quantity competition, they find a unique pure-strategy SPNE with

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simultaneous production in the first period. For price competition, there are two pure-strategy SPNE with endogenous leadership. The second of HS' models is a game of timing with action commitment, which makes leadership possible only if a firm produces first and commits to its quantity. In this model, they find two pure-strategy SPNE with endogenous leadership and a pure-strategy SPNE with simultaneous production, independently of whether competition is in prices or in quantities.[1]

A natural extension of HS' models is to consider asymmetric information on market demand. This is done by Mailath (1993) for the action commitment model. According to this model, the better-informed firm has a chance to decide its timing of movement, thereby avoiding signaling distortion. He shows that the unique equilibrium which survives the divinity criterion (Cho and Kreps (1987)) in this model is the one where the better-informed firm produces first. Normann (2002) discusses the role of asymmetric information in HS' observable delay model. Both endogenous leadership and simultaneous production appear as equilibria of this model.

A different approach to endogenous timing can be found in Maggi (1996), who analyzes a two-period investment game in a new market, which is modeled through a stochastic profitability parameter. Although the framework considered here is not a production oligopoly as above, the essential strategic situation is similar. The distinguishing features of the model are as follows. First, the uncertainty about profits is automatically resolved at the beginning of the second period; that is, a firm obtains complete information by just waiting. Second, firms can invest in both periods and let their own investments accumulate. *Ex ante* (i.e. before the actual state is observed by both firms), the equilibrium involves sequential play, with one firm committing to its first-period investment and the other waiting to invest later. *Ex post* (i.e. after the uncertainty is resolved), the eventually realized investments of both firms may be symmetric or asymmetric, depending on the degree of uncertainty and the difference between the expected and realized profitability of the market.

We briefly single out a few other relevant contributions to the endogenous timing literature. Pal (1998) studies the endogenous timing problem of a mixed oligopoly, meaning an oligopoly with a welfare-maximizing public firm and several profit-maximizing private firms. Equilibrium configurations depend on the number of private firms and time periods, but under no circumstances can a profile where all firms move simultaneously be a SPNE. A related paper from Lu (2006) introduces foreign competitors into the endogenous timing model for a mixed oligopoly. Finally, Ishibashi (2008) discusses the endogenous

[1] The action commitment model was revisited by van Damme and Hurkens (1999, 2004), who refined the model's equilibria on the basis of risk dominance considerations. Using the tracing procedure, they found that the equilibrium in which the more efficient firm behaves as Stackelberg leader is risk dominant, regardless of whether competition is in prices or in quantities.

timing problem for collusive price leadership with capacity constraints. He shows that the strategy profile in which one firm sets price first and the other firms follow the same price can be a SPNE. The larger the capacity of the price leader, the easier it is to sustain this SPNE.

In this paper, I explore firms' timing choices in a quantity-setting duopoly model with stochastic demand and costly market research. The basic structure of the model is based on HS' game of timing with action commitment. There are, however, two important differences. First, firms producing in the first period (or not engaging in market research) will only have imperfect information on the market demand. Second, both firms can update their information by carrying out (costly) market research.

The timing choices of both firms depend on a trade-off. Each firm can either carry out market research in the first period to obtain accurate market information, or produce first aiming to obtain a "first mover advantage". We will show that the qualitative characteristics of the subgame-perfect Nash equilibria are determined by the combined effects of market conditions (measured by the expectation and variance of market capacity), technology (measured by production costs), and market research costs. If the market research cost K is too high relative to the variance of market capacity, market research never plays a role. The intuition is simply that it does not pay to eliminate the uncertainty. For an intermediate value of K, we find two SPNE with endogenous leadership, independently of production costs. This is a relatively clear-cut case, in which one firm takes the trade-off and receives more information but becomes a follower, and the other firm obtains the first-mover advantage but pays the price of facing an uncertain demand.

If K is low enough (relative to the variance of market capacity), the situation is more complex. For given market conditions, there are two SPNE exhibiting endogenous leadership, provided the production cost of both firms are low enough. If the production cost of one firm is low enough but that of the other one is very high, then there is a unique SPNE with the more efficient firm as a leader. If production costs of both firms are too high, there is a SPNE involving simultaneous production in the second period. If market conditions becomes more favorable (higher expectation and lower variance of the market capacity), the SPNE with endogenous leadership survives even if the production of the leader is less efficient. The appearance of the SPNE with simultaneous production also requires much higher production costs of both firms. The converse is true if market conditions are unfavorable.

2 The Basic Model

We consider a quantity-setting duopoly in a market with stochastic demand. The inverse demand function is given by $P(Q) = a - Q$ (i.e. we normalize the slope of market demand to 1). Market capacity, given by the parameter $a > 0$, is a random variable with support contained in an interval $[a^L, a^H]$, assumed to have expectation $E[a]$ and variance $V[a] \neq 0$. In particular, a might be a continuous random variable or take only a finite number of values. Firm i ($i = 1, 2$) has a constant marginal cost c_i, which satisfies $0 < c_1 \leq c_2 < a^L$. To simplify the analysis, we also assume that $c_2 < \frac{1}{3}(2a^L - E[a] + 2c_1)$ to ensure that all the quantities used in the model are strictly positive.[2]

There are two time periods. [3] The (unknown) market demand does not change during the two periods, or, equivalently, it is realized at the end of the second period. At the beginning of the first period, firms have prior beliefs on a as stated above and can choose among three different choices: (i) to produce a certain quantity; (ii) to carry out market research; and (iii) to wait.

If a firm decides to produce a certain quantity in the first period, it observes neither the realized market demand nor the choice of its opponent. We assume that production is final, that is, a firm which produces in the first period can not produce additional units in the second period. Note that if a firm gives up market research, its information on market demand in the second period will remain the same as in the first period, and therefore there is no informational incentive to change the produced quantity in the second period.[4]

If a firm decides to carry out market research, it will find out the realization of market capacity at a cost $K \geq 0$, and observe the first period's choice of its opponent. In the second period, the firm will choose a quantity with full knowledge of the market demand. We say that this firm has perfect information on market demand. If a firm decides simply to wait, it observes its opponent's first-period choice, but cannot update its market information. In the second period, it has to choose a quantity without additional knowledge of the market

[2]This inequality guarantees the perfect-information Stackelberg follower quantity of firm 2 to be positive. It also ensures that all other quantities used in the analysis are positive.

[3]A period is treated as an indivisible time unit in which firms can start and finish their actions. A detailed discussion about the effect of time length on strategic timing choices is in Pacheco-de-Almeida and Zemsky (2003), where each period is further subdivided in T subunits.

[4]Thus, in this model, firms producing in the first period *commit* to a quantity. Commitment issues can of course be discussed at length, but are orthogonal to the issue of endogenous timing. Henkel (2002) studies the issue of commitment in a model of alternating moves (i.e. exogenous timing) where a player announces a decision and fixes a deviation cost; this player can revise the initial decision after a second player acts, by paying the deviation cost. In equilibrium, the chosen cost becomes a device to make the commitment credible. If actions are strategic substitutes, in the unique SPNE player 1 announces the Stackelberg leader action and a large deviation cost, player 2 responds with the Stackelberg follower action, and player 1 does not revise his action. Player 1 obtains the so-called "1.5th mover advantage".

demand. We say that this firm has imperfect information on market demand.

In short, information on the market demand is updated only after market research and each firm produces in one period only. Thus our model is quite different from two-period production models as Saloner (1987), where firms can accumulate outputs in two periods. It is an extension of HS' game of timing with action commitment allowing for stochastic market demand and market research.

Formally, the model gives rise to an extensive-form game. The set of players is $I = \{1, 2\}$. For each firm $i \in \{1, 2\}$, let M_i denote the choice of carrying out market research and W_i the choice of waiting in the first period. The action of firm i in the first period is denoted by

$$s_i^1 \in S_i^1 = \mathbb{R}^+ \cup \{M_i, W_i\},$$

and the second period decision is given by a mapping

$$s_i^2 : ([a^L, a^H] \cup \{W_i\}) \times (\mathbb{R}^+ \cup \{M_{-i}, W_{-i}\}) \to \mathbb{R}^+.$$

For example, $s_i^2(a|q)$ is the output level of firm i in period two, given that in the first period this firm carried out market research revealing market capacity a, but its opponent produced q. Analogously, $s_i^2(W_i|M_{-i})$ is the output level of firm i in period two, given that in the first period this firm decided to wait, but its opponent carried out market research.

Denote by S_i^2 the set of all functions s_i^2 as above. The strategy set of firm i is given by $S_i = S_i^1 \times S_i^2$, with typical element $s_i = (s_i^1, s_i^2)$. The payoff function of firm i is $\pi_i : S_i \times S_{-i} \to \mathbb{R}^+$.

Figure 1 shows the extensive-form of this game for the particular case where the random variable a follows a Bernoulli distribution, with $a = a^H$ with probability p and $a = a^L$ with probability $1 - p$.

In studying the extensive-form game, we make the modelling decision to have nature move at the beginning of the second period. This does not change the economic model at all, but generates an extensive-form game, in which every possible strategic situation in the second period corresponds to a proper subgame. Thus the appropriate equilibrium concept is simply Subgame-Perfect Nash Equilibrium (SPNE). An alternative treatment following Harsanyi's transformation would be to have nature move at the beginning of the first period. Under such a setup, many second-period situations would fail to give rise to proper subgames and we would have to resort to the use of Perfect Bayesian Equilibrium (PBE) as a solution concept. This added complication is void of economic content. A PBE consists of a belief system and a strategy profile, but the belief system in this model is trivial, since the beliefs on market capacity are exogenously given. Given these beliefs,

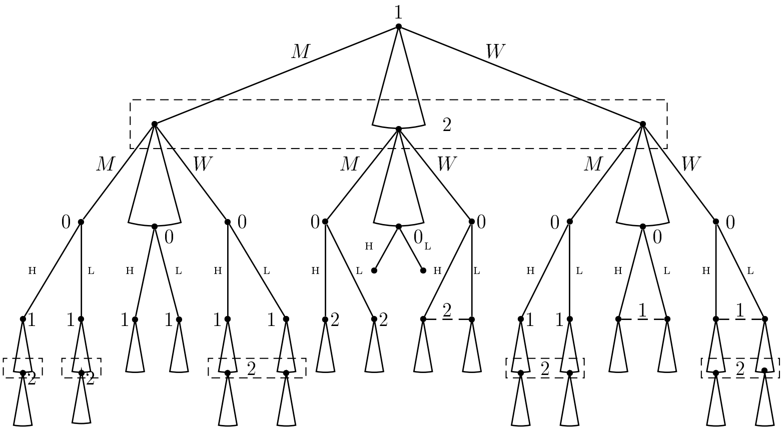

Figure 1: The extensive form when a can only take two values. Dashed lines and boxes indicate information sets.

one would use sequential rationality to derive the equilibrium strategy profiles, which are identical with those of the SPNE in the first treatment. Thus our choice allows us to greatly simplify the notation.

3 Equilibrium Analysis

3.1 Equilibrium Behavior in the Second Period

We first determine both firms' decisions in the second period; that is, we find the Nash Equilibria in each proper subgame.

Two informed firms. If both firms choose market research in the first period, the second-period subgames (one for each possible realization of a) are such that both firms produce simultaneously with perfect information on the market demand. The Nash equilibrium of one of these subgames corresponds to the Cournot-Nash equilibrium of the perfect-information duopoly. The equilibrium quantity of each firm i for each state $a \in [a^L, a^H]$ is denoted by $q_i^c(a)$. that is, the equilibrium strategy must prescribe

$$s_i^2(a|M_{-i}) = q_i^c(a) = \frac{1}{3}(a - 2c_i + c_{-i}),\ i = 1, 2 \tag{1}$$

which is always a strictly positive quantity.[5]

Two uninformed firms. If both firms choose to wait, neither of them is informed on the realized market demand. This leads to a second-period (Bayesian) subgame where both firms produce simultaneously with imperfect information on the market demand. The Nash equilibrium corresponds to the Cournot-Nash equilibrium of the imperfect-information duopoly, where the equilibrium quantity of each firm i is given by $q_i^c(E[a])$. Thus, the equilibrium strategies are such that

$$s_i^2(W_i|W_{-i}) = q_i^c(E[a]) = \frac{1}{3}(E[a] - 2c_i + c_{-i}),\ i = 1, 2. \tag{2}$$

One informed firm. If firm i choose to carry out market research and firm $-i$ to wait, in the second-period subgame both firms produce simultaneously with asymmetric information. In equilibrium, the informed firm conditions on the realization of a, but the uninformed firm does not. Denote the equilibrium quantity of the informed firm by $q_i^{Ic}(a)$. It is easy to show that

$$s_i^2(a|W_{-i}) = q_i^{Ic}(a) = \frac{1}{6}(3a - E[a] - 4c_i + 2c_{-i}), \forall\ a \in [a^L, a^H],\ i = 1, 2 \tag{3}$$

which is always strictly positive.[6] From the point of view of the uninformed firm $-i$, the expected equilibrium output of firm i is $q_i^c(E[a])$. Thus the equilibrium quantity of the uninformed firm is equal to $q_{-i}^c(E[a])$. Hence, in equilibrium

$$s_i^2(W_i|M_{-i}) = q_i^c(E[a]),\ i = 1, 2. \tag{4}$$

A leader and an informed follower. If firm i chose to carry out market research and firm $-i$ to produce a certain quantity, in the corresponding second-period subgame only firm i plays, choosing a certain quantity knowing both the demand and the quantity

[5] For the more efficient firm's quantity $q_1^c(a)$, this follows from $0 < c_1 \leq c_2 < a^L$. For the other firm, $q_2^c(a) > 0\ \forall\ a$ if $c_2 < \frac{1}{2}(a^L + c_1)$. This condition follows from the assumption $c_2 < \frac{1}{3}(2a^L - E[a] + 2c_1)$, because $\frac{1}{2}(a^L + c_1) - \frac{1}{3}(2a^L - E[a] + 2c_1) = \frac{1}{6}(2E[a] - a^L - c_1) > 0$.

[6] For the more efficient firm, $q_1^{Ic}(a) > 0$ if $c_1 < \frac{1}{4}(3a^L - E[a] + 2c_2)$. This follows from the assumptions $c_1 \leq c_2 < \frac{1}{3}(2a^L - E[a] + 2c_1)$ and the fact that $\frac{1}{3}(2a^L - E[a] + 2c_1) - \frac{1}{4}(3a^L - E[a] + 2c_2) = -\frac{1}{12}[(6c_2 - 6c_1) + (a^L + E[a] - 2c_1)] < 0$. For the less efficient firm, $q_2^{Ic}(a) > 0$ if $c_2 < \frac{1}{4}(3a^L - E[a] + 2c_1)$. This follows from the assumption $c_2 < \frac{1}{3}(2a^L - E[a] + 2c_1)$, because $\frac{1}{3}(2a^L - E[a] + 2c_1) - \frac{1}{4}(3a^L - E[a] + 2c_1) = -\frac{1}{12}(a^L + E[a] - 2c_1) < 0$.

of its opponent. The equilibrium strategy of firm i is thus to adopt a best response to its opponent's quantity. That is,

$$s_i^2(a|s_{-i}^1) = \max\{0, \frac{1}{2}(a - c_i - s_{-i}^1)\} \ , \forall\ s_{-i}^1 \in \mathbb{R}^+,\ i = 1, 2. \tag{5}$$

A leader and an uninformed follower. If firm i chose to wait and firm $-i$ to produce a certain quantity, in the corresponding second-period subgame only firm i plays, choosing a certain quantity knowing the quantity of its opponent but not the demand. The equilibrium strategy is

$$s_i^2(W_i|s_{-i}^1) = \max\{0, \frac{1}{2}(E[a] - c_i - s_{-i}^1)\}, \forall\ s_{-i}^1 \in \mathbb{R}^+,\ i = 1, 2. \tag{6}$$

Lemma 1 summarizes our computations.

Lemma 1. *In any SPNE, if a firm $i = 1, 2$ decides not to produce in the first period, in the second period its action s_i^2 must be such that*

$$s_i^2(a, s_{-i}^1) = \begin{cases} \frac{1}{3}(a - 2c_i + c_{-i}) & \text{if} \quad s_{-i}^1 = M \\ \frac{1}{6}(3a - E[a] - 4c_i + 2c_{-i}) & \text{if} \quad s_{-i}^1 = W \\ \frac{1}{2}(a - c_i - s_{-i}^1) & \text{if} \quad s_{-i}^1 \in \mathbb{R}^+ \end{cases} \tag{7}$$

for all $a \in [a^L, a^H]$, and

$$s_i^2(W_i, s_{-i}^1) = \begin{cases} \frac{1}{3}(E[a] - 2c_i + c_{-i}) & \text{if} \quad s_{-i}^1 = M \ or\ W \\ \frac{1}{2}(E[a] - c_i - s_{-i}^1) & \text{if} \quad s_{-i}^1 \in \mathbb{R}^+ \end{cases} \tag{8}$$

3.2 Equilibrium Behavior in the First Period

Let $\bar{s}_i^2 \in S_i^2$ be the functions defined in Lemma 1. Taking them as given (i.e. applying backwards induction), the extensive-form game can be simplified to a reduced normal-form game in which both firms only have to decide (in the first period) whether to produce, carry out market research, or wait. We would like to emphasize again that "produce in the first period" is not a single choice, but merely a simplified expression that we use to indicate that the firm chooses some quantity in $\mathbb{R}^+$. Hence in the reduced normal-form game, the choice set of each firm is $\mathbb{R}^+ \bigcup \{M_i, W_i\}$. Table 1 shows the expected payoffs for a firm[7] in this reduced normal-form game, where q_i stands for a quantity in $\mathbb{R}^+$ for both $i \in \{1, 2\}$.

[7]Since the game is symmetric, only firm i's payoff is shown in the table.

	q_{-i}	M_{-i}	W_{-i}
q_i	$(E[a]-q_i-q_{-i}-c_i)q_i$	$\frac{1}{2}(E[a]-q_i-2c_i+c_{-i})q_i$	$\frac{1}{2}(E[a]-q_i-2c_i+c_{-i})q_i$
M_i	$\frac{1}{4}E[(a-c_i-q_{-i})^2]-K$	$\frac{1}{9}E[(a-2c_{-i}+c_i)^2]-K$	$\frac{1}{36}E[(3a-E[a]-4c_i+2c_{-i})^2]-K$
W_i	$\frac{1}{4}(E[a]-c_i-q_{-i})^2$	$\frac{1}{9}(E[a]-2c_i+c_{-i})^2$	$\frac{1}{9}(E[a]-2c_i+c_{-i})^2$

Table 1: The Reduced Normal-Form Game

We use this reduced normal-form game to find the possible pure-strategy equilibria.[8] We proceed as follows. First we find equilibrium candidates, e.g., assuming that there is an equilibrium where both firms produce in the first period, we determine what the optimal production plans should be. Later on we will check whether these candidate equilibria are actually equilibria, e.g., whether firms have an incentive to deviate to either wait or carry out market research.

Both firms research. When both firms choose to carry out market research, the equilibrium candidate corresponds to the Cournot-Nash equilibrium quantities with perfect information. In each state, each firm i will produce $q_i^c(a)$ and receive net profits $\pi_i^c(a)-K$, where

$$\pi_i^c(a)=\frac{1}{9}(a-2c_i+c_{-i})^2, \forall\ i\in\{1,2\} \tag{9}$$

Then the expected net profit of firm i is $E[\pi_i^c(a)]-K$.

Both firms produce. When both firms choose production, the unique equilibrium candidate is the profile where each firm produces $q_i^c(E[a])$. Expected profits are then $\pi_i^c(E[a])$, that is,

$$\pi_i^c(E[a])=\frac{1}{9}(E[a]-2c_i+c_{-i})^2. \tag{10}$$

Both firms wait. When both firms choose to wait, the equilibrium candidate again involves the Cournot-Nash equilibrium quantities with imperfect information. Each firm i will produce $q_i^c(E[a])$ and receive expected profits $\pi_i^c(E[a])$. The only difference with the previous case is that these quantities are actually produced in the second period.

Production vs. Research. When firm i chooses to produce and firm $-i$ carries out market research, the unique equilibrium candidate corresponds to the Stackelberg equilibrium where the follower has superior information. We denote the equilibrium quantity of

[8]In mixed-strategy equilibria, firms randomize between producing first, carrying out market research, and waiting. Thus, there are no clear-cut timing choices which could be used to explain the appearance of endogenous leadership or simultaneous production.

the uninformed leader by q_i^ℓ and that of the informed follower by $q_{-i}^f(a)$.

$$q_i^\ell = \frac{1}{2}(E[a] - 2c_i + c_{-i}), \tag{11}$$
$$q_{-i}^f(a) = \frac{1}{4}(2a - E[a] - 3c_{-i} + 2c_i). \tag{12}$$

We also denote π_i^ℓ the expected profit of the leader and $\pi_{-i}^f(a)$ the gross profit of the informed follower in state a. That is,

$$\pi_i^\ell = \frac{1}{8}(E[a] - 2c_i + c_{-i})^2, \tag{13}$$
$$\pi_{-i}^f(a) = \frac{1}{16}(2a - E[a] - 3c_{-i} + 2c_i)^2. \tag{14}$$

Hence the expected net profit of the informed follower $-i$ is $E[\pi_{-i}^f(a)] - K$.

Production vs. Waiting. When firm i chooses to produce and firm $-i$ to wait, the equilibrium candidate corresponds to the Stackelberg equilibrium with imperfect information. The uninformed Stackelberg leader will produce q_i^ℓ and receive expected profits π_i^ℓ. The uninformed follower will produce $q_{-i}^f(E[a])$ and receive expected profits $\pi_{-i}^f(E[a])$. That is,

$$\pi_{-i}^f(E[a]) = \frac{1}{16}(a - 3c_{-i} + 2c_i)^2 \tag{15}$$

Research vs. Waiting. When firm i carries out market research and firm $-i$ chooses to wait, the informed firm i will produce $q_i^{Ic}(a)$ for each state $a \in \{1, 2\}$ and firm $-i$ will produce $q_{-i}^c(E[a])$. The gross profit of the informed firm in each state is denoted by $\pi_i^{Ic}(a)$.

$$\pi_i^{Ic}(a) = \frac{1}{36}(3a - E[a] - 4c_i + 2c_{-i})^2 \tag{16}$$

The expected net profit of this firm is $E[\pi_i^{Ic}(a)] - K$. The expected profit of the uninformed firm is equal to $\pi_{-i}^c(E[a])$.

Table 2 shows the payoffs of the equilibrium candidates.

Now we can use the reduced normal-form game in Table 1 to check whether the nine equilibrium candidates enumerated above are actually Nash equilibria. By Lemma 1, the NE of the reduced normal-form game give rise to the SPNE of the extensive-form game. Of course, the structure of the set of NE depends crucially on the market research cost K. The following three subsections discuss the cases with large, small, and intermediate K, respectively.

	q_2	M_2	W_2
q_1	$\pi_1^c(E[a]), \pi_2^c(E[a])$	$\pi_1^\ell, E[\pi_2^f(a)] - K$	$\pi_1^\ell, \pi_2^f(E[a])$
M_1	$E[\pi_1^f(a)] - K, \pi_2^\ell$	$E[\pi_1^c(a)] - K, E[\pi_2^c(a)] - K$	$E[\pi_1^{Ic}(a)] - K, \pi_2^c(E[a])$
W_1	$\pi_1^f(E[a]), \pi_2^\ell$	$\pi_1^c(E[a]), E[\pi_2^{Ic}(a)] - K$	$\pi_1^c(E[a]), \pi_2^c(E[a])$

Table 2: The Payoffs of Equilibrium Candidates

3.3 High Market Research Costs ($K > \frac{1}{4}V[a]$)

Clearly, the incentives for carrying out market research decrease as K increases. Intuitively, if the market research cost is high enough, it will offset the gains from obtaining accurate market information. A cutoff value is derived from the following Proposition, above which waiting is always better than market research in the reduced normal-form game.

Proposition 1. *When $K > \frac{1}{4}V[a]$, conducting market research is strictly dominated by waiting in the reduced normal-form game, for both firms $i \in \{1, 2\}$. There are three pure-strategy SPNE: in one of the equilibria, both firms produce the Cournot-Nash equilibrium quantities in the first period. In the other two equilibria, firms behave as Stackelberg leader and follower respectively.*

The proofs of all propositions and theorems are relegated to the Appendix.

The fact that the cutoff value is related to the variance of market capacity is very intuitive, since the variance is a natural measure of the value of the information obtained through market research. According to Proposition 1, when $K > \frac{1}{4}V[a]$, the strictly dominated strategy, market research, can be eliminated in the reduced normal-form game. The model then becomes almost the same as HS' game of timing with action commitment, except for the stochastic market demand. Since firms are unable to update their information, each firm has the same strategic incentives as in a perfect information context. Hence, the equilibria are the same as those of the HS' action commitment model.[9] This is the less interesting case for our analysis, because market research plays no role.

3.4 Low Market Research Costs ($K < \frac{1}{9}V[a]$)

A lower market research cost K increases the likelihood that a firm will carry out market research rather than waiting. In the extreme case $K = 0$, clearly no firm would strictly

[9]Strictly speaking, HS' deterministic model is encompassed in our model as the particular case $V[a] = 0$. This means any market research cost $K > 0$ is "large".

prefer waiting if market research is feasible. In the next Proposition, we find a cutoff value for K, below which carrying out market research always outperforms waiting.

Proposition 2. *When $K < \frac{1}{9}V[a]$, waiting is strictly dominated by market research in the reduced normal-form game, for both firms $i \in \{1, 2\}$.*

Again, the fact that the cutoff value is related to the variance of market capacity is very intuitive. For a large variance, information is very valuable and hence market research pays off.

Proposition 2 greatly simplifies the analysis whenever $K < \frac{1}{9}V[a]$. In this case, we can eliminate the strictly dominated strategy, waiting, in the reduced normal-form game. In order to find out the pure-strategy NE in the reduced normal-form game, only four equilibrium candidates remain. Table 3 shows the payoffs of these candidates.

	q_2	M_2
q_1	$\pi_1^c(E[a])$, $\pi_2^c(E[a])$	$\pi_1^\ell, E[\pi_2^f(a)] - K$
M_1	$E[\pi_1^f(a)] - K,\ \pi_2^\ell$	$E[\pi_1^c(a)] - K, E[\pi_2^c(a)] - K$

Table 3: The Payoffs of Equilibrium Candidates for $K < \frac{1}{9}V[a]$

The next Theorem summarizes the results.

Theorem 1. *When $K < \frac{1}{9}V[a]$, for any subgame-perfect Nash equilibrium $(s_i^1, s_i^2)_{i=1,2}$, second-period decisions are given by $s_i^2 = \bar{s}_i^2$ as in Lemma 1, and first-period decisions are:*

(i) (q_1^ℓ, M_2) *if and only if* $c_1 \leq \frac{1}{2}(c_2 + E[a] - \beta)$*;*

(ii) (M_1, q_2^ℓ) *if and only if* $c_2 \leq \frac{1}{2}(c_1 + E[a] - \beta)$*;*

(iii) (M_1, M_2) *if and only if* $c_1 \geq \frac{1}{2}(c_2 + E[a] - \beta)$*;*

where $\beta = 2\sqrt{2}\sqrt{V[a] - 9K}$. Further, the condition in (ii) implies the condition in (i).

We now briefly discuss this result. First note that the Theorem implies that, for $K < \frac{1}{9}V[a]$, the equilibrium candidate where both firms produce the imperfect-information Cournot-Nash equilibrium quantity $q_i^c(E[a])$ in the first period cannot give rise to a SPNE. The reason is that for each firm i, the deviation from $q_i^c(E[a])$ to M_i changes the firm's expected profit from $\pi_i^c(E[a])$ to $E[\pi_i^{Ic}(a)] - K$. When $K < \frac{1}{9}V[a]$, this deviation pays off. In other words, the gains from market research offset the cost K.

Item (i) identifies the necessary and sufficient condition for the strategy profile, where firm 1 chooses to produce the Stackelberg leader quantity and firm 2 chooses to carry out market research in the first period, to be a SPNE. Clearly, firm 2 (the follower) will not deviate to any other quantities in the second period if market research is chosen, since $\bar{s}_2^2$ prescribes the optimal output level. Nor will it deviate to producing in the first period, because the gains from market research, i.e. the expected gross profit of being a follower minus the expected profit from a first-period best response against the Stackelberg leader quantity of firm 1,

$$E[\pi_2^f(a)] - \pi_2(q_2^f(E[a])|q_1^\ell),$$

offsets the market research cost K in this case. On the other hand, firm 1 (the leader) will not deviate from q_1^ℓ if it has chosen to produce in the first period. The inequality in (i) guarantees that it will also not deviate to carry out market research in order to form a perfect-information Cournot duopoly with firm 2. This inequality implies that the production cost of firm 1 should be low enough. The intuition is simply that firm 1's leadership entails a low production cost to pay the price of market uncertainty and prevent the deviation to low-cost market research. However, the more favorable market conditions are, the less efficient the leader must be for the condition to be fulfilled. To see this, simply note that the inequality in (i) implies that the maximal production cost of firm 1 that supports this SPNE is increasing in $E(a)$ but decreasing in $V[a]$.

The reason for this last observation is simple. Given $V[a]$, the increment of π_1^ℓ induced by $E[a]$ is larger than the increment of $E[\pi_1^c(a)] - K$, the expected net profit of firm 1 when deviating to carrying out market research.[10] If $E[a]$ is large enough, the expected profit gained through leading the market will be higher than that from market research. Given $E[a]$, firm 1's incentive to deviate from producing first diminishes as $V[a]$ becomes smaller, because a low $V[a]$ indicates a relatively stable market capacity (low uncertainty). For small $V[a]$, firm 1's prior belief in the market demand, without added market research, already enables firm 1 to earn a higher profit than that following market research.

In short, firm 1 prefers producing first, given that its opponent conducts market research, whenever its production is efficient enough, relative to market conditions, for the "first mover advantage" to dominate the "informational advantage" of market research. As a result, endogenous leadership with an efficient leader and an informed inefficient follower

[10] Technically, $\frac{\partial \pi_i^\ell}{\partial E[a]} = \frac{1}{4}(E[a] + c_j - 2c_i)$ and $\frac{\partial}{\partial E[a]}(E[\pi_1^c(a)] - K) = \frac{2}{9}(c_j - 2c_i)$. If $i = 1$, then $\frac{\partial \pi_i^\ell}{\partial E[a]} - \frac{\partial}{\partial E[a]}(E[\pi_1^c(a)] - K) = \frac{2}{9}E[a] + \frac{1}{36}[(E[a] - c_1) + (c_2 - c_1)] > 0$. If $i = 2$, then $\frac{\partial \pi_i^\ell}{\partial E[a]} - \frac{\partial}{\partial E[a]}(E[\pi_1^c(a)] - K) = \frac{1}{36}c_1 + \frac{1}{18}(E[a] - c_2) + \frac{7}{36}E[a] > 0$. When $E[a] > 2c_i - c_{-i} + 2\sqrt{\frac{1}{5}(V[a] - 9K)}$, π_i^ℓ is always larger than $(E[\pi_i^c(a)] - K)$.

appears in the equilibrium path.

Item (ii) shows the necessary and sufficient condition for the converse situation to the one in (i) to be a SPNE, that is, endogenous leadership with an inefficient leader and an informed efficient follower. Firm 2 (the inefficient firm) produces the Stackelberg leader quantity and firm 1 carries out market research in the first period. The analysis is analogous to that for (i). The follower will not deviate for the same reason given for (i). The leader will not deviate either, if its marginal cost is small enough. The only difference is that, for given c_1 and c_2, market conditions need to be more favorable for the inefficient firm to assume the leader role than for the efficient one. This is implicit contained in the inequality in (ii). Comparing it to the inequality in (i), one finds that, *ceteris paribus*, it entails a higher $E[a]$ or a lower $V[a]$. The reason is simply that firm 2, without carrying out market research, will suffer a higher loss than firm 1 for given market conditions, simply because it is less efficient. In other words, whenever an endogenous-leadership SPNE with an inefficient leader exists, there is also a SPNE with an efficient leader.

The SPNE with simultaneous production in the second period appears if the condition in item (iii) is fulfilled. This condition requires the marginal costs of both firms to be high enough, relative to market conditions. The reason is that inefficient firms will suffer large losses due to market uncertainty, hence both firms would like to carry out market research. It should be pointed out that for very unfavorable market conditions, even if both firms have relatively low costs, no firm will produce in the first period. To see this explicitly, note that the inequality implies that the minimal production cost of firm 1 that supports this SPNE is increasing in $E[a]$ and decreasing in $V[a]$. For unfavorable market conditions, the information about demand becomes so important that both firms would like to investigate the market and assume both the market research cost K and the ensuing harsher competition (firms become Cournot duopolist forgoing the possibility to become Stackelberg leaders). That is, "informational advantage" dominates the "first mover advantage."

Figure 2 illustrates Theorem 1 using a numerical example. It shows the areas where the three possible SPNE exist, in the coordinate system of marginal costs for $K < \frac{1}{9}V[a]$. In this example, a follows a Bernoulli distribution taking the value $a = 10$ with probability 0.7 and the value $a = 20$ otherwise. The line through $OCEF$ represents the function $c_1 = c_2$. Since we assume $c_1 \leq c_2$, the relevant area is the triangle OFH. In this region, the line through CDG is the function $c_1 = (c_2 + E[a] - \beta)$. The line through AC shows the function $c_2 = (c_1 + E[a] - \beta)$. The line through BDE shows the function $c_2 = \frac{1}{3}(2a^L - E[a] + 2c_1)$. According to Theorem 1, the strategy profile in which only firm 1 produces first is a SPNE if c_1 and c_2 fall in the $OCDB$ region. In the area OAC, the strategy profile in which only

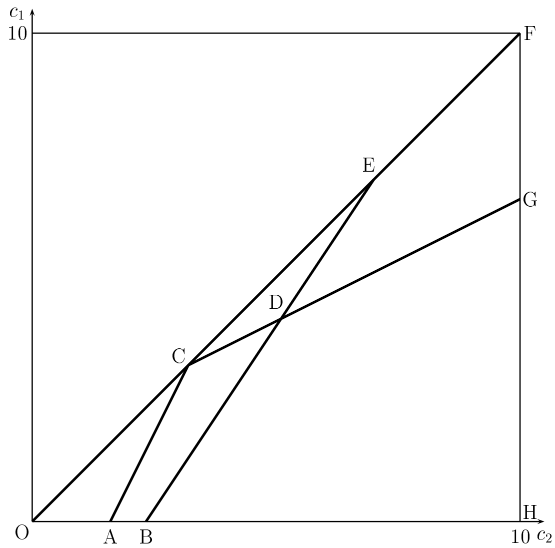

Figure 2: A numerical example for $K < \frac{1}{9}V[a]$. In this example, a is assumed to follow a Bernoulli distribution; that is with a probability of 0.7, $a = 10$ and with a probability of 0.3, $a = 20$. Hence $E[a] = 13$ and $V[a] = 21$. The market research cost is assumed to be 1.

firm 2 produces first is a SPNE. In the CDE region, the strategy profile where both firms carry out market research is a SPNE. We see that if c_1 and c_2 are such that an SPNE with an inefficient leader exists, then the existence of an SPNE with an efficient leader follows automatically.

3.5 Intermediate Market Research Costs ($\frac{1}{9}V[a] \leq K \leq \frac{1}{4}V[a]$)

We now turn to the case of intermediate market research costs, i.e. $\frac{1}{9}V[a] \leq K \leq \frac{1}{4}V[a]$. In this case, no strategy is strictly dominated in the reduced normal-form game, thus we need to discuss all nine equilibrium candidates. We first explore the case with strict inequalities, $\frac{1}{9}V[a] < K < \frac{1}{4}V[a]$. This rules out the situations where waiting is weakly dominated by market research ($K = \frac{1}{9}V[a]$) and where market research is weakly dominated by waiting ($K = \frac{1}{4}V[a]$). Theorem 2 lists the SPNE for this scenario.

Theorem 2. *When $\frac{1}{9}V[a] < K < \frac{1}{4}V[a]$, for any pure-strategy subgame-perfect Nash* *equilibrium* $(s_i^1, s_i^2)_{i=1,2}$*, second-period decisions are given by* $s_i^2 = \bar{s}_i^2$ *as in Lemma 1, and the first-period decision is either* (q_1^ℓ, M_2) *or* (M_1, q_2^ℓ)*, for any* $c_1 \leq c_2$*.*

According to Theorem 2, two SPNE with endogenous leadership appear independently of the production costs. The leader, who produces q_i^ℓ, will not deviate to waiting, because giving up leadership without obtaining accurate market information never pays off. Nor will it deviate to market research. The proof of Theorem 2 in the Appendix shows that the market research cost in this case is high enough to ensure that the deviation to market research is not worthwhile. The follower also has no incentive to deviate. As long as $K < \frac{1}{4}V[a]$, market research generates a higher profit than waiting, regardless of the quantity produced by the other firm in the first period (shown in the proof of Proposition 2). The follower has no incentive to deviate to producing the best reply to the leader's quantity in the first period either. The reason is that the gain from carrying out market research is the expected gross profit of the informed follower minus the expected profit of the uninformed follower, and this difference is larger than an intermediate market research cost (see inequality (27)).

The equilibrium candidate with simultaneous production in the first period is not a SPNE, as long as $K < \frac{1}{4}V[a]$. Given the imperfect-information Cournot-Nash equilibrium quantity of the opponent, one firm can benefit from market research, even though the market research cost is relatively high. We would also like to emphasize that, for $K > \frac{1}{9}V[a]$, none of the four equilibrium candidates with simultaneous production in the second period can be a NE. Neither (M_1, M_2) nor (W_1, W_2) can give rise to a SPNE, since both firms have an incentive to deviate to producing the Stackelberg leader quantity in the first period (shown in the proof of Theorem 2). Finally, (W_i, M_{-i}) $\forall$ $i \in \{1, 2\}$ cannot give rise to a SPNE, because as shown in the proof of Proposition 1 (equation (19)), the firm choosing market research always has incentive to deviate to wait when $K > \frac{1}{9}V[a]$.

Figure 3, generated by the same example as in Figure 2, illustrates the last result. All lines in this figure represent the same functions as in Figure 2. For instance, the line through BE shows the function $c_2 = \frac{1}{3}(2a^L - E[a] + 2c_1)$ in the region $c_1 \leq c_2$. The two SPNE with endogenous leadership appear in the area OBE.

Consider now the knife-edge scenarios where $K = \frac{1}{9}V[a]$ or $K = \frac{1}{4}V[a]$. In both cases, we have payoff ties and hence firms are indifferent among two choices. For $K = \frac{1}{9}V[a]$, all the SPNE in Theorems 1 and 2 remain valid. Further, in this case $\pi_i(W_i|M_{-i}) = \pi_i(M_i|M_{-i}), \forall i \in \{1, 2\}$ and it follows that (W_i, M_{-i}) $\forall$ $i \in \{1, 2\}$ are equilibria, which give rise to two SPNE with simultaneous production in the second period. For $K = \frac{1}{4}V[a]$, all the SPNE in Proposition 1 and Theorem 2 remain valid.

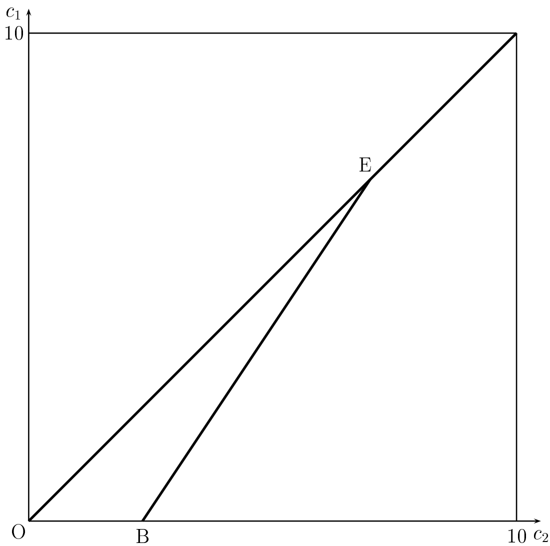

Figure 3: A numerical example for $\frac{1}{9}V[a] < K < \frac{1}{4}V[a]$.

4 Conclusion

Real-world firms always face a certain degree of uncertainty about market demand. This creates an incentive for market research. However, this has not been adequately taken into account in the existing literature on firms' endogenous timing choices. In previous literature, firms either face a deterministic market demand (e.g. HS), have asymmetric information (Mailath (1993) and Normann (2002)), or can automatically update information without any cost (Maggi (1996)). This paper develops an endogenous timing model, which explains the appearance of endogenous leadership or simultaneous production when market research plays a role. The model is an extension of HS' action commitment model allowing for stochastic market demand and (costly) information updating. The original model is encompassed in the present model as the particular case with $V[a] = 0$.

The model provides a new explanation for the appearance of endogenous leadership or simultaneous production. Namely, market research induces a trade-off between the "informational advantage" and the "first mover advantage", which eventually determines firms' timing choices. If a firm decides to produce in the first period, it has a chance to obtain the "first mover advantage" but lose the "informational advantage". The converse is true for a firm carrying out market research and producing later. As a result, the model provides a better prediction than HS' action commitment model. In the latter, both kinds of endogenous leadership and simultaneous production appear in the equilibrium; thus the model lacks predictive power.

In contrast, our model delivers clear-cut predictions in a variety of parameter constellations. When the market research cost is small, there is a unique SPNE with simultaneous production in the second period, provided production costs of both firms are high and market capacity has a small expectation and large variance. For the same level of market research costs, if the production cost of the efficient firm is small enough but that of inefficient one is large, there is a unique pure-strategy SPNE with the efficient firm as endogenous leader, as long as market capacity has a relatively small expectation and large variation, so that only the efficient firm can be the leader. For intermediate values of K, simultaneous production does not appear in equilibrium; only two pure-strategy SPNE with endogenous leadership exist, independently of production costs.

Finally, we find a somewhat counter-intuitive result. Firms might forgo market research even with market uncertainty and low market research costs, $K \leq \frac{1}{9}V[a]$. In this case, firms are willing to face market uncertainty and produce in the first period in order to obtain the "first mover advantage".

Appendix

Proof of Proposition 1

Proof. Let $K > \frac{1}{4}V[a]$. We want to show that market research is strictly dominated by waiting in the reduced normal-form game. Denote by $\pi_i(\cdot|\cdot)$ the payoffs in this game (as given in Table 1).

Suppose first that firm $-i$ chooses some output level $s^1_{-i} \in \mathbb{R}^+$ in the first period. Firm i strictly prefers W_i rather than M_i if and only if

$$
\begin{aligned}
& \pi_i(W_i|q^1_{-i}) > \pi_i(M_i|q^1_{-i}) \\
\Leftrightarrow \quad & \frac{1}{4}\left(E[a] - s^1_{-i} - c_i\right)^2 > E\left[\frac{1}{4}(a - s^1_{-i} - c_i)^2\right] - K \\
\Leftrightarrow \quad & K > \frac{1}{4}V[a] \qquad (17)
\end{aligned}
$$

Second, if firm $-i$ decides to wait, W_i is strictly better than M_i for firm i if and only if

$$\begin{aligned}
& \pi_i(W_i|W_{-i}) > \pi_i(M_i|W_{-i}) \\
\Leftrightarrow \quad & E\left[\frac{(E[a]+c_{-i}-2c_i)^2}{9}\right] > \frac{1}{36}E\left[(3a-E[a]-4c_i+2c_{-i})^2\right]-K \\
\Leftrightarrow \quad & K > \frac{1}{4}V[a] \qquad (18)
\end{aligned}$$

Last, if firm $-i$ conducts market research, then for firm i, W_i is strictly preferred to M_i if and only if

$$\begin{aligned}
& \pi_i(W_i|M_{-i}) > \pi_i(M_i|M_{-i}) \\
\Leftrightarrow \quad & \frac{1}{9}(E[a]+c_i-2c_{-i})^2 > E\left[\frac{(a+c_{-i}-2c_i)^2}{9}\right]-K \\
\Leftrightarrow \quad & K > \frac{1}{9}V[a] \qquad (19)
\end{aligned}$$

Combining conditions (17), (18) and (19), if $K > \frac{1}{4}V[a]$, market research is strictly dominated by waiting (for both firms) in the reduced normal-form game. □

Proof of Proposition 2

Proof. The proof follows form conditions (17), (18) and (19), simply reversing all inequalities. It follows that, if $K < \frac{1}{9}V[a]$, waiting is strictly dominated by market research (for both firms) in the reduced normal-form game. □

Proof of Theorem 1

Proof. We look for all the pure-strategy Nash equilibria for $K < \frac{1}{9}V[a]$ in the reduced normal-form game. We only need to check which of the four equilibrium candidates, shown in Table 3, are actually Nash equilibria.

Both firms produce $q_i^c(E[a])$ in the first period. When each firm uses this strategy, the expected profit of firm i is $\pi_i^c(E[a])$. Since waiting is strictly dominated by market research according to Proposition 2, we only need to check whether a deviation to M_i would pay off. This deviation would enable firm i to adopt the best response to its opponent's quantity $q_{-i}^c(E[a])$ in each state $a \in [a^L, a^H]$,

$$BR_i(a, q_{-i}^c(E[a])) = q_i^{Ic}(a) = \frac{1}{6}(3a - E[a] + 2c_{-i} - 4c_i) \qquad (20)$$

Hence the expected net profit of firm i would be $E[\pi_i^{Ic}(a)] - K$. Thus, $(q_i^c(E[a]), q_{-i}^c(E[a])$ is a NE if and only if $\pi_i^c(E[a]) \geq E[\pi_i^{Ic}(a)] - K$ for both $i \in \{1, 2\}$. That is,

$$\begin{aligned}
& \pi_i^c(E[a]) - E[\pi_i^{Ic}(a)] + K \geq 0 \\
\Leftrightarrow\ & \frac{1}{9}(E[a] + c_{-i} - 2c_i)^2 - \frac{1}{36}E[(3a - E[a] + 2c_{-i} - 4c_i)^2] + K \geq 0 \\
\Leftrightarrow\ & K \geq \frac{1}{4}V[a] \qquad (21)
\end{aligned}$$

Therefore, when $K < \frac{1}{9}V[a]$, both firms have incentive to deviate to market research. It follows that $(q_i^c(E[a]), q_{-i}^c(E[a])$ is not a NE in the reduced normal-form game.

Firm 1 produces q_1^ℓ and firm 2 chooses M_2 in the first period. The profits of firm 1 when producing q_1^ℓ are π_1^ℓ. Again, the best deviation of firm 1 is to choose M_1, which, given that firm 2 chooses M_2, results in a perfect-information Cournot duopoly in the second period and generates the expected net profit $E[\pi_1^c(a)] - K$ for firm 1. Therefore, firm 1 will have no incentive to deviate from q_1^ℓ to M_1 if and only if $\pi_1^\ell \geq E[\pi_1^c(a)] - K$. That is,

$$\begin{aligned}
& \frac{1}{8}(E[a] - 2c_1 + c_2)^2 \geq E\left[\frac{1}{9}(a - 2c_1 + c_2)^2\right] - K \\
\Rightarrow\ & 9(E[a] - 2c_1 + c_2)^2 - 8E[(a - 2c_1 + c_2)^2] + 72K \geq 0 \\
\Rightarrow\ & (2c_1 - c_2)^2 - 2E[a](2c_1 - c_2) + E[a]^2 \geq 8V[a] - 72K \\
\Rightarrow\ & (2c_1 - c_2 - E[a])^2 \geq 8(V[a] - 9K) \qquad (22)
\end{aligned}$$

For $K \leq \frac{1}{9}V[a]$, the last inequality holds if and only if

$$c_1 \leq \frac{1}{2}\left(E[a] + c_2 - 2\sqrt{2}\sqrt{V[a] - 9K}\right) \quad \text{or} \qquad (23)$$

$$c_1 \geq \frac{1}{2}\left(E[a] + c_2 + 2\sqrt{2}\sqrt{V[a] - 9K}\right) \qquad (24)$$

Inequality (24) implies that $c_1 > c_2$, which contradicts the assumption that $c_1 \leq c_2$. Thus, inequality (23) is the necessary and sufficient condition for firm 1 not to have an incentive to deviate from q_1^ℓ.

The expected net profit of firm 2 when choosing M_2 is $E[\pi_2^f(a)] - K$. If firm 2 deviates to producing in the first period, the optimal quantity is given by

$$BR_2(q_1^\ell) = q_2^f(E[a]) = \frac{1}{4}(E[a] - 3c_2 + 2c_1) \qquad (25)$$

which generates the expected profit

$$\pi_2(q_2^f(E[a])|q_1^\ell) = \frac{1}{16}(E[a] - 3c_2 + 2c_1)^2 \tag{26}$$

Hence, firm 2 will have no incentive to deviate if and only if

$$\begin{aligned} & E[\pi_2^f(a)] - K \geq \pi_2(q_2^f(E[a])|q_1^\ell) \\ \Leftrightarrow \quad & E\left[\frac{(2a - E[a] - 3c_2 + 2c_1)^2}{16}\right] - K - \frac{1}{16}(E[a] - 3c_2 + 2c_1)^2 \geq 0 \\ \Leftrightarrow \quad & K \leq \frac{1}{4}V[a] \end{aligned} \tag{27}$$

Since we assume $K < \frac{1}{9}V[a]$ here, firm 2 will not deviate from M_2. Hence, when $K < \frac{1}{9}V[a]$, the strategy profile (q_1^ℓ, M_2) is a NE in the reduced normal-form game if and only if (23) is satisfied.

Firm 2 produces q_2^ℓ and firm 1 carries out market research in the first period. This situation is analogous to the previous one. Firm 2 (the leader) will not have an incentive to deviate if $\pi_2^\ell \geq E[\pi_2^c(a)] - K$. When $K < \frac{1}{9}V[a]$, this condition holds if and only if

$$c_2 \leq \frac{1}{2}\left(E[a] + c_1 - 2\sqrt{2}\sqrt{V[a] - 9K}\right) \quad \text{or} \tag{28}$$

$$c_2 \geq \frac{1}{2}\left(E[a] + c_1 + 2\sqrt{2}\sqrt{V[a] - 9K}\right) \tag{29}$$

Inequality (29) implies that $c_2 > \frac{1}{2}(E[a] + c_1)$, which contradicts our assumption that $c_2 < \frac{1}{3}(2a^L - E[a] + 2c_1) < \frac{1}{2}(E[a] + c_1)$ (it would imply a negative $q_2^f(E[a])$). Hence, inequality (28) is the necessary and sufficient condition for firm 2 not to deviate from q_2^ℓ.

Firm 1 (the follower) will not have an incentive to deviate if its profits as a follower are larger than or equal to the profit from taking a best reply against q_2^ℓ without market research, i.e. if $\pi_1^\ell \geq \pi_1(q_2^f(E[a])|q_2^\ell)$. This condition holds if and only if $K \leq \frac{1}{4}V[a]$. Thus when $K < \frac{1}{9}V[a]$ firm 1 will not deviate. The strategy profile (M_1, q_2^ℓ) is a NE in the reduced normal-form game if and only if condition (28) is satisfied.

Both firms carry out market research in the first period. In this case, the expected profit of each firm i is $E[\pi_i^c(a)] - K$. The best deviation of firm i, given its opponent chooses M_{-i}, is to produce q_i^ℓ in the first period. Firm i will not have an incentive to deviate if

$E[\pi_i^c(a)] - K \geq \pi_i^\ell$ for each $i \in \{1,2\}$. That is,

$$
\begin{aligned}
& E\left[\frac{1}{9}(a - 2c_i + c_{-i})^2\right] - K \geq \frac{1}{8}(E[a] - 2c_i + c_{-i})^2 \\
\Rightarrow \quad & 8E[(a - 2c_i + c_{-i})^2] - 72K - 9(E[a] - 2c_i + c_{-i})^2 \geq 0 \\
\Rightarrow \quad & (2c_i - c_{-i} - E[a])^2 \leq 8(V[a] - 9K) \qquad (30)
\end{aligned}
$$

For $K < \frac{1}{9}V[a]$, this condition holds if and only if

$$
\begin{aligned}
\frac{1}{2}\left(E[a] + c_2 - \beta\right) \leq \quad c_1 \quad & \leq \frac{1}{2}\left(E[a] + c_2 + \beta\right) \quad \text{and} \qquad (31) \\
\frac{1}{2}\left(E[a] + c_1 - \beta\right) \leq \quad c_2 \quad & \leq \frac{1}{2}\left(E[a] + c_1 + \beta\right) \qquad (32)
\end{aligned}
$$

The RHS of equation (31) is (weakly) larger than that of equation (32), because $c_1 \leq c_2$. Recalling our assumption on the positivity of all relevant quantities (i.e. $c_2 < \frac{1}{3}(2a^L - E[a] + 2c_1)$), and the fact that $\frac{1}{3}(2a^L - E[a] + 2c_1) - \frac{1}{2}(E[a] + c_1) = -\frac{1}{6}(4a^L + c_1 - 5E[a]) \leq 0$, we see that $c_2 \leq \frac{1}{2}\left(E[a] + c_1 + \beta\right)$ is automatically fulfilled. It is also easy to verify that the LHS of equation (31) is (weakly) larger than that of equation (32). Since $c_1 \leq c_2$, we conclude that (31) and (32) are fulfilled if and only if

$$
c_1 \geq \frac{1}{2}\left(E[a] + c_2 - \beta\right).
$$

□

Proof of Theorem 2

Proof. We want to prove that, for $\frac{1}{9}V[a] < K < \frac{1}{4}V[a]$, the only SPNE correspond to the two equilibrium candidates with endogenous leadership in the reduced normal-form game, independently of c_1 and c_2.

Suppose firm i produces q_i^ℓ and firm $-i$ chooses M_{-i} in the first period, where $i \in \{1,2\}$. Firm $-i$ will not have an incentive to deviate to producing in the first period as long as its profits as informed follower are larger than or equal to the profits from adopting a best response to q_i^ℓ in the first period, i.e. $E[\pi_{-i}^f(a)] - K \geq \pi_{-i}(q_{-i}^f(E[a])|q_i^\ell)$. As shown in (27) in the proof of Theorem 1, this inequality holds if and only if $K \leq \frac{1}{4}V[a]$. The proof of Proposition 1 shows that, for $K < \frac{1}{4}V[a]$, market research always generates higher profits than wait, for any output level of the leader. Hence, firm $-i$ has no incentive to deviate to W_{-i} either.

Firm i will not have an incentive to deviate from q_i^ℓ to M_i if its expected profits as leader are larger than or equal to the expected net profit generated by market research, i.e. if $\pi_i^\ell \geq E[\pi_i^c(a)] - K$. This condition is always fulfilled for $K \geq \frac{1}{9}V[a]$. To see this, note that the expected profit difference

$$\pi_i^\ell - (E[\pi_i^c(a)] - K) = \frac{1}{8}[E[a] + c_{-i} - 2c_i]^2 - \frac{1}{9}E[(a + c_{-i} - 2c_i)^2] + K \tag{33}$$

is an increasing function of K and, for $K = \frac{1}{9}V[a]$, attains the value

$$\pi_i^\ell - E[\pi_i^c(a)] = \frac{1}{72}(E[a] - 2c_i + c_{-i})^2 \geq 0 \tag{34}$$

independently of c_i $\forall\ i \in \{1, 2\}$. Thus firm i has no incentive to deviate to market research for $\frac{1}{9}V[a] < K < \frac{1}{4}V[a]$. It will not deviate to wait if $\pi_i^\ell \geq \pi_i^c(E[a])$ either, because

$$\pi_i^\ell = \frac{1}{8}(V[a] - 2c_i + c_{-i})^2 < \frac{1}{9}(V[a] - 2c_i + c_{-i})^2 = \pi_i^c(E[a]). \tag{35}$$

We conclude that the strategy profiles (q_i^ℓ, M_{-i}) for both $i \in \{1, 2\}$ are NE of the reduced-form game.

None of the strategy profiles with simultaneous production in the second period is a NE for $\frac{1}{9}V[a] < K < \frac{1}{4}V[a]$. Let us start with (M_1, M_2). As shown in (33) and (34) above, $E[\pi_i^c(a)] - K < \pi_i^\ell$ for $K > \frac{1}{9}V[a]$. Thus both firms have an incentive to deviate to q_i^ℓ. The profile (W_1, W_2) is not a NE either, because by (35) above firms have an incentive to deviate to producing q_i^ℓ. The profiles (W_i, M_{-i}) $\forall\ i\ \in \{1, 2\}$ are not NE either, because the waiting firm has an incentive to deviate to q_i^ℓ, since $\pi_i^\ell > \pi_i^c(E[a])$ by (35).

Finally, we have to prove that $(q_i^c(E[a]), q_{-i}^c(E[a]))$ is not a NE for $\frac{1}{9}V[a] < K < \frac{1}{4}V[a]$. This is immediate from (21) in the proof of Theorem 1, which implies that deviating to market research pays off if $K < \frac{1}{4}V[a]$. □

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