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Strategic Reasoning in Hide-and-Seek Games: A Note

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Strategic Reasoning in Hide-and-Seek Games: A Note*

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Abstract

Aggregate behavior in two-player hide-and-seek games deviates systematically from the mixed-strategy equilibrium prediction of assigning all actions equal probabilities (Rubinstein and Tversky, 1993, Rubinstein et al., 1996, Rubinstein, 1999). As Crawford and Iriberri (2007) point out, this deviation can be explained by strategic level-k reasoning. Here we provide empirical evidence that, indeed, it is non-equilibrium beliefs that lead to the behavior observed in the earlier studies: when a player's opponent is forced to play the equilibrium strategy, the player's choices are uniformly spread over the action space. At the same time, we find robust evidence of an unexpected framing effect.

Keywords: *Salience, level-k reasoning, cognitive hierarchy, hide-and-seek game, framing effect*
JEL-Classification: C72, C91

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1 Introduction

Game theory aims at predicting behavior in strategic interactions. As experimental studies have shown, game-theoretic predictions are often accurate in the sense that behavior converges to these predictions over time. However, first-round behavior in repeated games and behavior in one-shot games often differs considerably from game-theoretic predictions. One reading of this fact is that a different approach for characterizing and predicting inexperienced and one-shot play is needed.[1]

The level-k model of strategic reasoning is one such approach. It assumes that people differ with respect to their level of reasoning. Level-0 types react instinctively to the game without reasoning about others' incentives, while level-k types with $k > 0$ best-respond to the strategy employed by level-(k-1) types. This model has been stimulated by the observations from the guessing game (Nagel, 1995; Ho, Camerer and Weigelt, 1998) but has also been applied successfully to many other games (e.g., Stahl and Wilson, 1994, 1995; Costa-Gomes, Crawford and Broseta, 2001; Costa-Gomes and Crawford, 2006; Crawford and Iriberri, 2007b).

A recent application by Crawford and Iriberri (2007a) provides an explanation for a puzzle posed by data from the experiments on hide-and-seek games by Rubinstein and Tversky (1993), Rubinstein, Tversky and Heller (1996) and Rubinstein (1999). In these matching-pennies-like games both players employ a mixed strategy in equilibrium, assigning all actions equal probabilities. Hence, aggregate behavior should be described best by a uniform distribution over all possible choices. However, when implemented experimentally, aggregate behavior differs from this prediction in a systematic way. Using an econometric approach, Crawford and Iriberri (2007a) show that level-k play provides a plausible explanation for the deviation in Rubinstein and coauthors' data.

In this paper, we seek to provide behavioral evidence for this explanation, using the fact that level-k players differ from traditional game-theoretic agents only in their non-equilibrium beliefs. In our experiment, we fix participants' beliefs by forcing their opponent overtly to play the equilibrium strategy. If behavior differs between our experiment and the experiments run by Rubinstein and coauthors, we can infer that participants' beliefs are different, and thus, the claim that players held out-of-equilibrium beliefs in the original experiments is supported. The results from our main treatments show that this is the case.

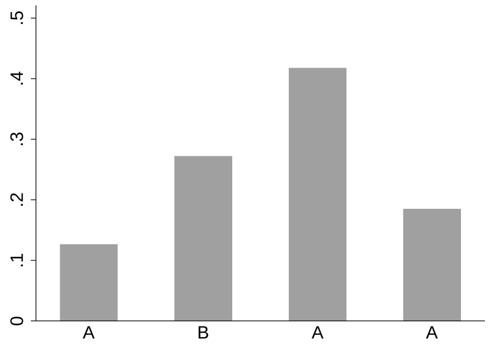
[1] See Crawford et al. (2010) for a survey of recent approaches.

2 Existing empirical evidence

In their experimental studies, Rubinstein and Tversky (1993), Rubinstein, Tversky and Heller (1996), and Rubinstein (1999) (RTH in the following) present their subjects with hide-and-seek games under differently-labeled action sets. In their most prominent setup, they present the game as two opponents facing a row of four boxes that are labeled as "A", "B", "A", and "A". The hider places a prize in one of the boxes and the seeker, not having observed the hider's choice, may open one of the boxes in order to find it. If their choices match, the seeker gets the prize, otherwise the hider keeps it.

While standard game theory would predict the labels not to influence subjects' behavior, RTH's data suggest otherwise. RTH explain this influence by the labels' salience. According to their interpretation, the "B" location is salient because it is the only "non-A"; the "A"s at the two ends because of their marginal location. This leaves the third box, "central A", as the least salient location. RTH find that at least 40 % of hiders and even more seekers choose "central A", leaving seekers (hiders) much better (worse) off than under the equilibrium prediction. Figure 1 presents the pooled choice frequencies of hiders from Rubinstein, Tversky and Heller (1996) and Rubinstein (1999).

Figure 1: *Choice frequencies of hiders observed by* RTH *(pooled)*



In their explanation of the results, RTH conclude that "the players employed a naïve strategy (avoiding the endpoints), that is not guided by valid strategic reasoning" (Rubinstein, Tversky, and Heller, 1996, p. 402). Crawford and Iriberri (2007) propose a different explanation and show that a level-k approach is able to account for the findings if it is based on a level-0 that follows Rubinstein et al.'s assessment of salience. For their preferred level-k model, the econometric analysis yields no evidence for level-0 play. In fact, they observe that subjects are using higher levels of reasoning than in most other games.

Table 1: *Treatments (Sessions)*

	Beliefs	
	Equilibrium	Unrestricted
Framing		
Original	EqB-Orig (Dec 2009, 76 participants) (Dec 2010, 90 participants)	UnB-Orig (Oct 2011, 208 participants)
Neutral	EqB-Neut (Dec 2009, 82 participants) (Dec 2010, 110 participants)	UnB-Neut (Oct 2011, 214 participants)

3 Our experiment

In this paper, we introduce two treatment variations to the original experiments. The first is designed to test whether the non-equilibrium behavior documented in Rubinstein, Tversky and Heller (1996) and Rubinstein (1999) is a consequence of non-equilibrium beliefs as the level-k explanation suggests. For this purpose, we inform participants that we would assign their opponent an action by a random draw over the action set rather than let the opponent choose the action. In other words, participants face an opponent who is known to play the equilibrium strategy (the instructions stated that the opponent's choice would be "*randomly* determined for him").[2] While we do not make a prediction for behavior in this variation, we make an out-of-sample prediction: there is no reason for behavior in RTH's experiments to differ from that in our experiment, unless players' beliefs are different.[3] Hence, data that does not differ from RTH's data would suggest there are alternative explanations for the latter besides the level-k hypothesis. Data that differs from RTH's data—and especially behavior that cannot be distinguished from a uniform distribution—would lead us to conclude that players do not believe their opponent will play the equilibrium strategy in the original game. We refer to this variation as "equilibrium beliefs" (EqB) and to the original setup as "unrestricted beliefs" (UnB).

The instructions in our experiment followed the wording by Rubinstein

[2]See the appendix for a translated version of the first session's complete instructions. Afterwards we added a question to identify and exclude the few subjects who participated in more than one session.

[3]Recall that if players hold equilibrium beliefs in the original game, they are certain their opponent will mix uniformly, and hence, the two situations are equivalent.

(1999).[4] The second treatment variation is designed to check the robustness of the data to changes in the game's frame. For this variation we changed the original wording of "your opponent" and "selecting a box" to "the other player" and "hiding the prize in the box", respectively, thus avoiding the hide-and-seek frame without changing the incentives or the salience pattern. We refer to this variation as "neutral" framing (NEUT) and to the original wording as "original" framing (ORIG).

Table 1 summarizes the four resulting treatments. In all treatments first-year students of introductory economics played RTH's A-B-A-A game in a classroom setting at the University of Duisburg-Essen, Germany. The instructions were amended by adding that we would randomly draw two participants who would be paid and that the successful player would receive a prize of 50 Euros. To ensure anonymity, participants were identified by randomly generated identification codes. In the EQB treatments, we restricted the experiment to the role of hiders only, as this would allow us to answer the research question while maximizing the number of data points available. In this case, the first participant's decision was implemented, whereas the second participant's decision was determined by the publicly enacted draw of a colored ball from an urn by a volunteering student.

In a first run-through of the EQB treatments in December 2009, 158 students participated. To check for robustness of our results, we repeated the treatments one year later, in December 2010, with another cohort of 200 students. The data for the UNB treatments was collected in October 2011 with a third cohort of 667 students. 422 of them played the game as hiders while the remaining participants acted as seekers. In all sessions, participants were assigned randomly to the NEUT and ORIG conditions, which differed only with respect to the wording of the instructions.

4 Results

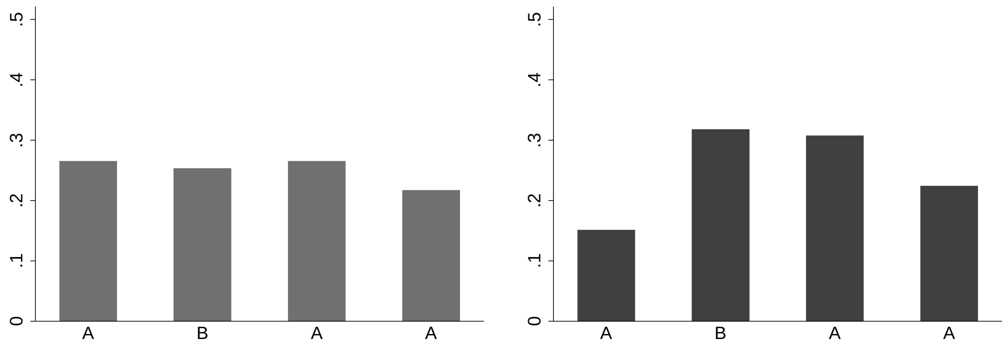
First of all, we do not find any evidence for a framing effect on hider behavior in hide-and-seek games with unrestricted beliefs ($p = 0.927$ for UNB-ORIG vs UNB-NEUT, 208 and 214 observations, respectively).[5] Thus, we pool the UNB-ORIG and UNB-NEUT data for further analysis. Comparing the data to RTH's data, we do not find evidence of a subject-pool effect, either ($p = 0.195$ individual-treatment comparisons to RTH's data yield $p = 0.207$ for UNB-ORIG and $p = 0.328$ for UNB-NEUT). Furthermore, we find no significant differences between the two sessions of our main treatments with

[4] See http://arielrubinstein.tau.ac.il/99/gt100.html.

[5] All p-values correspond to χ^2-tests.

induced equilibrium beliefs that were conducted with two different cohorts of fist-year students ($p = 0.437$ for EqB-Orig and $p = 0.699$ for EqB-Neut). Accordingly, we pool the data from 2009 and 2010 for our analysis.[6] The resulting distributions of choices are presented in Figure 2.

Figure 2: *Choice frequencies in* EqB-Orig *(left) and* EqB-Neut *(right)*



We make three noteworthy observations. First, when presenting our participants with instructions that closely follow the original instructions used by RTH (our EqB-Orig treatment), the choice distribution can no longer be distinguished from uniform randomization ($p = 0.793$). At the same time it is significantly different from RTH's data ($p = 0.013$) and our replication ($p = 0.010$; a comparison only to UnB-Orig yields $p = 0.070$).

Second, the response pattern is sensitive to the wording of the instructions: when the task is presented as one of "selecting boxes" as in our EqB-Neut treatment, the choice distribution is no longer uniform ($p = 0.003$). However, it cannot be distinguished statistically from the distribution observed by RTH ($p = 0.307$) or the replication ($p = 0.723$; the comparison to UnB-Neut only yields $p = 0.671$), even though the mode switches from "central A" to "B".[7]

Finally, note that we could have made the above claims with data from the 2009 session only (the according $p-$values for observations one and two are $p = 0.617$ and $p = 0.015$, respectively). However, we were not convinced our findings would be replicable. For this reason, we decided to replicate our earlier results and hold their robustness to be noteworthy.

[6]See the appendix for a detailed data overview.

[7]Intriguingly, we also observe a strong framing effect for the seekers in our replication ($p = 0.029$): in contrast to the typical distribution with a modal "central A" in UnB-Orig, the distribution in UnB-Neut is perfectly symmetric with one third of the density mass on "B", "central A", and the combination of the marginal "A"s, respectively. As a consequence, the seeker advantage relative to the equilibrium hit rate of 25% decreases from 27.74% in UnB-Orig to 26.71% in UnB-Neut.

5 Discussion

In their inspiring paper, Crawford and Iriberri (2007a) provide a convincing explanation for observations from a series of experiments on hide-and-seek games conducted by Rubinstein and Tversky (1993), Rubinstein, Tversky and Heller (1996) and Rubinstein (1999). They econometrically show that, contrary to the conjecture of Rubinstein and coauthors, the observed behavior can be explained by strategic behavior. In fact, their analysis suggests high levels of player sophistication in terms of a levels-of-reasoning approach.

In this note, we contribute to the discussion by providing empirical evidence that, indeed, non-equilibrium beliefs rather than naïve behavior are the culprit for the patterns observed in the original experiments: when hiders face an equilibrium player and instructions are close to the original instructions, choices are spread evenly over the action space (EqB-Orig). If hiders in the experiments of Rubinstein and his coauthors—or our replications UnB-Orig and UnB-Neut—believed their opponents to play as predicted by game theory, we have no reason to assume aggregate behavior to be any different from our data. Hence, if hiders in the original experiments had held equilibrium beliefs, our EqB-Orig treatment suggests their behavior would have been equilibrium behavior, too.

At the same time, our neutrally-worded EqB-Neut treatment poses a new puzzle that awaits a convincing explanation: in this treatment, responses concentrate around the middle, yet without the clear peak at "central A" reported in the original experiments. This could be due to the "pull-to-the-middle" effect documented in psychological questionnaire studies (Christenfeld, 1995; Shaw, Bergen, Brown and Gallagher, 2000; Attali and Bar-Hillel, 2003; Valenzuela and Raghubir, 2009), but it is far from clear why this effect would apply to the EqB-Neut treatment only, even though its "selecting-a-box" wording arguably is closer to a questionnaire than the original "hiding" variant. Also, there may be other factors playing a role, such as a stronger perception of simultaneity of moves in the EqB-Neut treatment. A fact that could be interpreted as an indication in this latter direction is that the seekers' advantage relative to the equilibrium prediction, both in rth's data and our UnB-Orig replication, is reduced in UnB-Neut; however, this interpretation contrasts with findings of Eliaz and Rubinstein (2011) who do not find sequence effects to be significant in the context of a repeated matching-pennies game. Yet, the findings from our EqB treatments are remarkable in that their robustness is a sign of *systematic* behavioral patterns: we find the same choice patterns in two different cohorts of first-year students.

All in all, our study highlights two points. First, when thinking about one-shot games, the traditional assumption of an "equilibrium in beliefs" is unwarranted. And second, it underlines the importance of the mindframe induced by the way a situation presents itself to an actor as well as the need for further research to provide an understanding of what these mindframes are and how they emerge.

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Appendix

Instructions for the EqB-Orig treatment

Part 1	Identification code: 34rk3qd

Welcome to the experiment!

You are participating in an analysis of decision making in the field of experimental economics. Please read the following instructions carefully and insert your answers to the questions. It is absolutely necessary that you make your decision independently. Unfortunately we have to exclude you from the experiment, if you communicate with others during the course of the experiment.

Instructions:

You and another student are playing the following game:

You possess a prize which you can hide in one of four boxes arranged in a row. The boxes are marked as follows: A, B, A, A.

Your opponent will be allowed to find the prize by opening one (and only one) of the four boxes. The choice of the box will be *randomly* determined for him. Where will you hide the prize so that it will remain in your possession? Please mark that box with a cross:

A	B	A	A

At the end of the lecture one participant, whose own decision will be relevant, will be randomly drawn. This participant will be matched with another randomly drawn participant, the opponent, whose decision will be determined by a random device. The successful player receives the prize of 50 Euro.

Additional information:

Please also provide the following information about yourself:

Age: _________ Semester: _________

Gender (please mark the appropriate box):

☐ female ☐ male

Field of study (please mark the appropriate box):

☐ business ☐ economics ☐ information systems ☐ teaching certificate ☐ other

Please cut off part 2 of this sheet now and keep it. You need it to receive your payoff in case that you are randomly drawn. Please fold part 1 once and wait until we start collecting part 1. Thank you for participating.

Part 2	Identification code: 34rk3qd

Instructions for the EqB-Neut treatment

Part 1	Identification code: 34rk3qd

Welcome to the experiment!

You are participating in an analysis of decision making in the field of experimental economics. Please read the following instructions carefully and insert your answers to the questions. It is absolutely necessary that you make your decision independently. Unfortunately we have to exclude you from the experiment, if you communicate with others during the course of the experiment.

<u>Instructions:</u>

You and another student are playing the following game:

You can choose one of four boxes arranged in a row. The boxes are marked as follows: A, B, A, A.

The other player will also choose one (and only one) of the four boxes. The choice of the box will be *randomly* determined for him. You will earn a prize if you select a different box than the other player. Otherwise the other player will earn the prize. Which box are you going to select in order to win the prize? Please mark that box with a cross:

A	B	A	A

At the end of the lecture one participant, whose own decision will be relevant, will be randomly drawn. This participant will be matched with another randomly drawn participant, the “other player”, whose decision will be determined by a random device. The successful player receives the prize of 50 Euro.

<u>Additional information:</u>

Please also provide the following information about yourself:

Age: _________ Semester: _________

Gender (please mark the appropriate box):

☐ female ☐ male

Field of study (please mark the appropriate box):

☐ business ☐ economics ☐ information systems ☐ teaching certificate ☐ other

Please cut off part 2 of this sheet now and keep it. You need it to receive your payoff in case that you are randomly drawn. Please fold part 1 once and wait until we start collecting part 1. Thank you for participating.

Part 2	Identification code: 34rk3qd

Data

Table 2: *Aggregate choice frequencies in the A-B-A-A game*

Session (participants; $p-$value of a χ^2-test under H_0: the preceding treatments have the same underlying data-generating process)	**Choice frequencies** (in %)			
	A	B	A	A
HIDERS				
Dec 2009, EqB-Orig (76)	30	20	26	24
Dec 2010, EqB-Orig (90)	23	30	27	20
EqB-Orig, pooled (166; $p = 0.437$)	**27**	**25**	**27**	**22**
Dec 2009, EqB-Neut (82)	13	35	32	20
Dec 2010, EqB-Neut (110)	16	29	30	25
EqB-Neut, pooled (192; $p = 0.699$)	**15**	**32**	**31**	**22**
Rubinstein, Tversky, and Heller (1996); hiders (53)	9	36	40	15
Rubinstein (1999); hiders (50)	16	18	45	22
rth, pooled (103; $p = 0.197$)	**13**	**27**	**42**	**18**
Oct 2011, UnB-Orig; hiders (208)	15	29	30	25
Oct 2011, UnB-Neut; hiders (214)	14	27	33	26
UnB, pooled (422; $p = 0.927$)	**15**	**28**	**32**	**26**
SEEKERS				
Rubinstein, Tversky, and Heller (1996); seekers (62)	8	19	28	7
Rubinstein (1999); seekers (64)	10	12	35	7
rth, pooled (126; $p = 0.466$)	**18**	**31**	**63**	**14**
Oct 2011, UnB-Orig; seekers (141)	13	31	72	25
Oct 2011, UnB-Neut; seekers (92)	15	31	31	15
UnB, not pooled (141 vs 92; $p = 0.029$)				

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